Name______________________________ #2 MVT, Rolle's, Extrema, and More!

1. Determine the critical points(s) for $g(x)=\sqrt{1-x^2}$.

$g'(x)=\frac{1}{2}(1-x^2)^{-1/2}(-2x)$

$\frac{-x}{\sqrt{1-x^2}}=0$ $=0$

$x=0$

$1-x^2=0$

$x^2=1$

$x=\pm 1$

$\boxed{g(0)=1 \quad g(-1)=0 \quad g(1)=0}$

2. Find the absolute extrema values for $h(x)=\frac{10}{x^2+1}$ for [-1, 2].

$h(x)=10(x^2+1)^{-1}$

$h'(x)=-10(x^2+1)^{-2}(2x)$

$h'(x)=\frac{-20x}{(x^2+1)^2}$ $=0$ $=0$ $\longrightarrow$ $x=0$

$x^2+1=0$ DNE

candidate test:

$h(-1)=5$

$h(0)=10$

$h(2)=2$

global min @ $(2,2)$

global max @ $(0,10)$

3. Verify that Rolle's Theorem does or does not apply to $f(x)=1-x^{2/3}$ for [-1, 1].
If it does apply, find c.

f is continuous ✓

f is not differentiable

$f(-1)=0$

$f(1)=0$

$f'(x)=0$

$-\frac{2}{3}x^{-1/3}=0$

$\frac{-2}{3\sqrt[3]{x}}=0$

f is not differentiable @ 0 (cusp). Therefore, Rolle's does not apply.

4. Let f be the function defined by $f(x)=x^3-x^2-2x$. What is the value of c for which the instantaneous rate of change of f at $x=c$ is the same as the average rate of change of f over [-1, 1].

$f'(c)=\frac{f(1)-f(-1)}{1--1}$

$3x^2-2x-2=\frac{-2-0}{2}$

$3x^2-2x-2=-1$

$3x^2-2x-1=0$

x^2-2x-3

$(x-\frac{3}{3})(x+\frac{1}{3})=0$

$(x-1)(x+\frac{1}{3})=0$

$x=1$ $x=-\frac{1}{3}$

$\boxed{c=-\frac{1}{3}, 1}$

5. For $y=3\ln(2x^3)$, find y'.

$y'=3\left[\frac{6x^2}{2x^3}\right]=\boxed{\frac{9}{x}}$

6. Water is flowing out at the rate of 6 m^3/min from a reservoir in the shape of a hemispherical bowl with radius 13 m. Given that the volume of water in a hemispherical bowl of radius R is $V=\frac{\pi}{3}y^2(3R-y)$ when the water is y units deep, how fast is the water level falling when the water is 8 m deep?

$\frac{dV}{dt}=-6\ m^3/min$

$r=13\ m$

Find $\frac{dy}{dt}$

$V=\frac{\pi}{3}y^2(3R-y)$

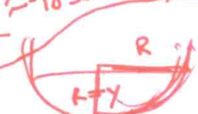
when $y=8$ $V=\frac{\pi}{3}y^2(3y-y)=\frac{\pi}{3}y^2 2y$

$V=\frac{2\pi}{3}y^3 \Rightarrow \frac{dV}{dt}=\frac{2\pi}{3}3y^2\frac{dy}{dt}$

$\frac{dV}{dt}=\frac{4\pi}{3}y\frac{dy}{dt}$

$-6=\frac{4\pi}{3}(13)\frac{dy}{dt}$

$\frac{dy}{dt}=\frac{-117}{2\pi}\approx -18.3783$ m/min



hemisphere $\Rightarrow$ R=Y

7. Let f be a differentiable functions with the following values given.

x	1	2	3	4	5
$f(x)$	-4	-1	1	6	13
$f'(x)$	3	4	2	5	6

Explain why there must be a value r with $3 < r < 5$ for which $f(r) = 10$.

Since f is differentiable, then it is continuous. Since it's continuous, and $f(3)=1$ and $f(5)=13$, by the IVT there must be some number "r" on [3,5] where $f(r)=10$.

Explain why there must be a value t with $3 < t < 5$ for which $f'(t) = 6$.

$\frac{f(5)-f(3)}{5-3} = \frac{13-1}{2} = 6$ Since f is differentiable, and $\frac{f(5)-f(3)}{5-3}=6$, by the MVT there exists some "t" where $f'(t)=6$.

8. Determine whether Rolle's Theorem is valid for $f(x) = 3 - |x - 2|$ on [-1, 5]. If so, find c. If not, tell why. f is not differentiable @ $x=2$, so Rolle's does not apply.

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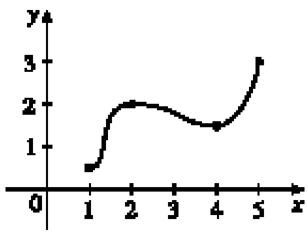
9.

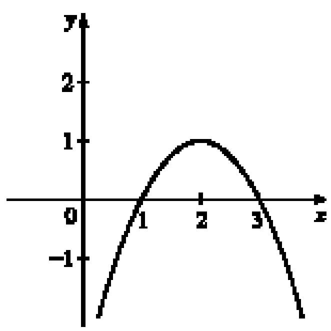
3. Absolute maximum at b; absolute minimum at d; local maxima at b and e; local minima at d and s; neither a maximum nor a minimum at a, c, r, and t.

10.

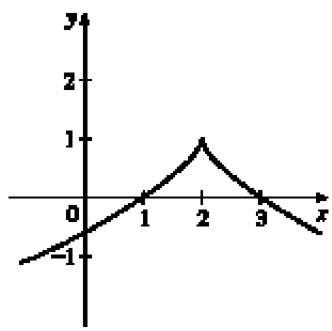
8. Absolute minimum at 1, absolute maximum at 5, local maximum at 2, local minimum at 4



11. (a)



(b)



(c)