Name___________________________

Riemann Sums

1. Suppose time, *t*, is given in seconds and your velocity, *v*, in $\frac{\text{m}}{\text{sec}}$, is given by

$$v(t) = 1 + t^2 \text{ for } 0 \le t \le 6,$$

Use $\Lambda t = 2$ to estimate the distance traveled during this time. Find the left-hand sum.

2. You jump out of an airplane. Before your parachute opens, you fall faster and faster but your acceleration decreases as you fall because of air resistance. The table gives you acceleration, *a* in m/s^2, after *t* seconds.

t	0	1	2	3	4	5
A	9.81	8.03	6.53	5.38	4.41	3.61

Give the upper estimate of your speed for the first 5 seconds if $n = 5$. Set up the solution, but you do not need to solve.

3. When an aircraft attempts to climb as rapidly as possible, it climbs rate decreases with altitude. (This occurs because the air is less dense at higher altitudes.) The table shows performance data for a certain single-engine aircraft.

Altitude (1000 ft)	0	1	2	3	4	5	6	7	8	9	10
Climb rate (ft/min)	925	875	830	780	730	685	635	585	535	490	440

Set up the Riemann sum for the midpoint estimate for the time required for this aircraft to climb from sea level to 10,000 ft if $n = 5$.

4. Use right-hand Riemann sums to estimate the total distance covered in 10 minutes. Set up the approximations, but no need to solve.

Time (minutes)	0	1.5	2	4	6.5	7	9	9.4	10
Velocity (ft/min)	120	180.6	241	360	480	500	515	605	650

5. A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time *t* minutes, where *v* is a differentiable function of *t*. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table below.

t(min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mi/min)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.2

a. Use data from the table to approximate $v'(18)$. Show the computations that lead to your answer. Using appropriate units, explain the meaning of your answer.

b. Using data from the table, determine the average rate of change of $v(t)$ over the interval $0 \le t \le 40$. Show the computations that lead to your answer.

c. Based on the values in the table, what is the smallest number of instances at which the velocity of the plane could be 8 miles per minute on the open interval $0 < t < 40$? Justify your answer.

d. Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could be 0 miles/min^2? Justify your answer.

e. Using data from the table and the best possible right Riemann sum, approximate the distance traveled by the plane over $0 \le t \le 40$. Show the computations that lead to your answer.

f. Using data from the table and the best possible midpoint Riemann sum, approximate the distance traveled by the plane over $0 \le t \le 40$. Show the computations that lead to your answer.

g. Using data from the table and the best possible trapezoidal sum, approximate the distance traveled by the plane over $0 \le t \le 40$. Show the computations that lead to your answer.

6. A table of values of $f(t)$ is given.

t	0	20	40	60	80	100	120
$f(t)$	1.2	2.8	4.0	4.7	5.1	5.2	4.8

(a) Estimate $\int_0^{120} f(t)$ by using a left Riemann sum with six subintervals.

(b) Estimate $\int_0^{120} f(t)$ by using a right Riemann sum with six subintervals.

(c) Estimate $\int_0^{120} f(t)$ by using a midpoint sum with three subintervals.

(d) Estimate $\int_0^{120} f(t)$ by using the trapezoidal rule with three subintervals.

7. A table of values of $g(t)$ is given.

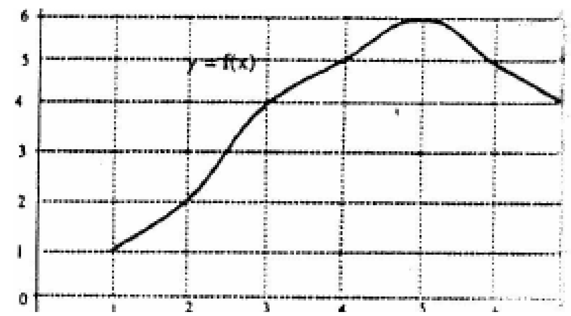
t	0	40	70	90	100
$g(t)$	150	180	195	184	172

(a) Estimate $\int_0^{100} g(t)$ by using a left Riemann sum with four subintervals.

(b) Estimate $\int_0^{100} g(t)$ by using a right Riemann sum with four subintervals.

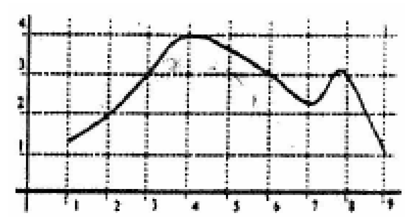
(c) Estimate $\int_0^{100} g(t)$ by using the trapezoidal rule with four subintervals.

8. The graph of the function *f* over the interval [1, 7] is shown. Using values from the graph, estimate $\int_1^7 f(x)dx$ using three trapezoids.



9. Use the same graph above, but use 6 trapezoids.

10. The graph of *f* over the interval [1, 9] is shown in the figure. Using the data, find a midpoint approximation with 4 equal subdivisions for $\int_1^9 f(x)dx$.



11. An experiment was performed in which oxygen was produced at a continuous rate. The rate at which oxygen was produced was measured each minute and the results tabulated.

Minutes	0	1	2	3	4	5	6
Oxygen (cu ft/min)	0	1.4	1.8	2.2	3.0	4.2	3.6

Use the trapezoid rule to estimate the amount of oxygen produced in 6 minutes.