Name________________________________ Class____________ Date____________________

Solids of Known Cross Section Worksheet

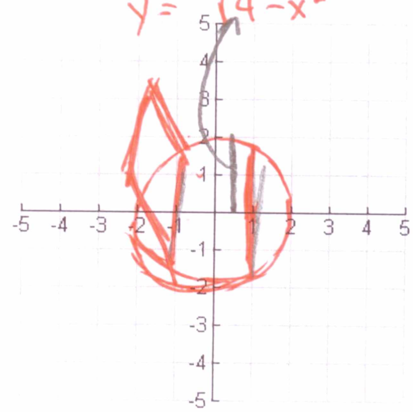
Sketch a graph of the base, draw a representative cross section, write a geometric formula for the area of the cross sectional face, set up a definite integral and find the volume.

1. Find the volume of the solid whose base is bounded by the circle defined by $x^2 + y^2 = 4$ whose cross sections perpendicular to the x-axis are squares.

r^2

$y^2 = 4 - x^2$

$y = \sqrt{4-x^2}$



$V = 2\int_0^2 \left(2\sqrt{4-x^2}\right)^2 dx$

$V = 8\int_0^2 (4-x^2)\,dx = 8\left[4x - \frac{1}{3}x^3\right]_0^2$

$V = 8\left[8 - \frac{8}{3} - (0)\right]$

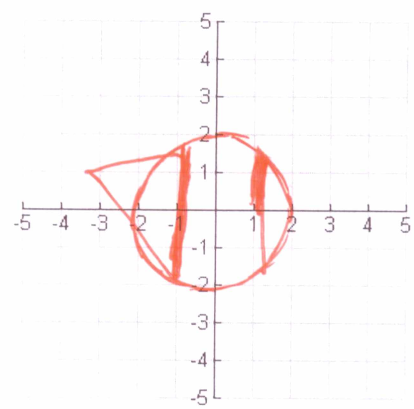
$V = 8\left(\frac{16}{3}\right)$

$V = \boxed{\frac{128}{3}}$

2. Find the volume of the solid whose base is bounded by the circle defined by $x^2 + y^2 = 4$ whose cross sections perpendicular to the x-axis are equilateral triangles.

$y = \sqrt{4-x^2}$

(The area of an equilateral triangle is given by $A = \frac{\sqrt{3}s^2}{4}$.



$V = 2\int_0^2 \frac{\sqrt{3}}{4}\left(2\sqrt{4-x^2}\right)^2 dx$

$V = 2\sqrt{3}\int_0^2 (4-x^2)\,dx$

$V = 2\sqrt{3}\left[4x - \frac{x^3}{3}\right]_0^2$

$V = 2\sqrt{3}\left[8 - \frac{8}{3} - (0)\right]$

$V = 2\sqrt{3}\left(\frac{16}{3}\right) = \boxed{\frac{32\sqrt{3}}{3}}$