Name Key

Calculus Test #6 Review

1. Find an equation of the tangent line to the graph of $y = x^2 + \ln(2x-5)$ at the point (3,9).

$y' = 2x + \frac{2}{2x-5}$

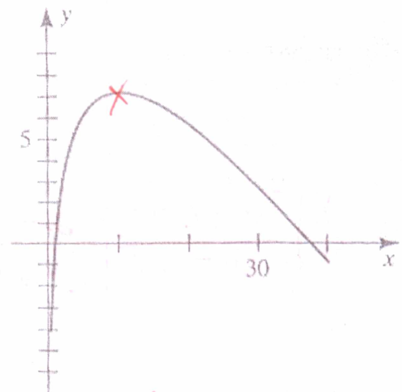
$m = 2(3) + \frac{2}{2(3)-5} = 6+2$

$m = 8$

$\boxed{y - 9 = 8(x-3)}$

2. Shown in the figure at the right is the graph of $y = 5\ln x - \frac{1}{2}x$.

Find the coordinates of the absolute/global maximum point on the interval (0,40].



$y' = \frac{5}{x} - \frac{1}{2} = 0$

$\frac{5}{x} = \frac{1}{2}$

$x = 10 \rightarrow y = 5\ln 10 - \frac{1}{2}(10) = 5\ln 10 - 5$

$\boxed{(10,\ 5\ln 10 - 5)}$

3. Find $f'(x)$ for $f(x) = \sqrt{1+e^{2x}} = (1+e^{2x})^{1/2}$

$f'(x) = \frac{1}{2}(1+e^{2x})^{-1/2}(e^{2x})(2)$

$= \frac{e^{2x}}{\sqrt{1+e^{2x}}}$

4. Find $f'(x)$ for $f(x) = \frac{e^x}{x^2+1}$

$f'(x) = \frac{(x^2+1)(e^x) - (e^x)(2x)}{(x^2+1)^2}$

$f'(x) = \frac{e^x(x^2+1-2x)}{(x^2+1)^2} = \frac{e^x(x^2-2x+1)}{(x^2+1)^2}$

$= \frac{e^x(x-1)^2}{(x^2+1)^2}$

5. Find an equation of the tangent line to the graph of $y = (x-1)e^x + 3\ln x + 2$ at the point (1,2).

$y' = (x-1)e^x + e^x$

$m = (1-1)e^1 + e^1 = e$

$\boxed{y - 2 = e(x-1)}$

6. Find $\frac{dy}{dx}$:

$xe^y + 2x - \ln(y+1) = 3$

$xe^y\frac{dy}{dx} + e^y + 2 - \frac{\frac{dy}{dx}}{y+1} = 0$

$\frac{dy}{dx}\left(xe^y - \frac{1}{y+1}\right) = -e^y - 2$

$\frac{dy}{dx} = \frac{-e^y-2}{xe^y - \frac{1}{y+1}} = \boxed{\frac{(y+1)(-e^y-2)}{xe^y(y+1)-1}}$

7. If $y = \frac{\ln x}{x}$, find y'.

$y' = \frac{x\left(\frac{1}{x}\right) - \ln x(1)}{x^2}$

$y' = \boxed{\frac{1-\ln x}{x^2}}$

8. Use logarithmic differentiation to evaluate the following: $y = \frac{e^{4x}-1}{e^{4x}+1}$.

$\ln y = \ln(e^{4x}-1) - \ln(e^{4x}+1)$

$\frac{1}{y}\frac{dy}{dx} = \frac{4e^{4x}}{e^{4x}-1} - \frac{4e^{4x}}{e^{4x}+1}$

$\frac{dy}{dx} = \left[\frac{4e^{4x}}{e^{4x}-1} - \frac{4e^{4x}}{e^{4x}+1}\right]\left[\frac{e^{4x}-1}{e^{4x}+1}\right]$