Name Key

Calculus Review #10

1. Evaluate the integral: $\int 6x dx$ $= 6\int x\,dx$

$= 6\left(\frac{x^2}{2}\right) + C$

$= 3x^2 + C$

2. Evaluate: $\int \frac{1}{h^4} dh$ $= \int h^{-4} dh$

$= \frac{h^{-3}}{-3} + C$ or $= \frac{1}{-3h^3} + C$

$\frac{10}{6}$

3. Evaluate: $\int \frac{4 + 5x^{\frac{5}{3}}}{\sqrt{x}} dx$ $= \int \left(\frac{4}{\sqrt{x}} + \frac{5x^{5/3}}{x^{1/2}}\right) dx$

$\frac{3}{6}$

$= 4\int x^{-1/2} dx + 5\int x^{7/6} dx$

$= 4\frac{x^{1/2}}{1/2} + 5\frac{x^{13/6}}{13/6}$

$= 8x^{1/2} + \frac{30}{13}x^{13/6} + C$

4. Find the particular solution of the equation $f'(x) = 2x^{-1/2}$ that satisfies the condition $f(1) = 6$.

$f(x) = \int 2x^{-1/2} = 2(2x^{1/2}) + C$

$f(x) = 4x^{1/2} + C$

$f(1) = 4(1)^{1/2} + C = 6$

$C = 2$

$\boxed{f(x) = 4\sqrt{x} + 2}$

5. Use $a(t) = -32 \text{ ft/s}^2$ as the acceleration due to gravity. A ball is thrown upward from the ground with an initial velocity of 56 feet per second. For how many seconds will the ball be going upward?

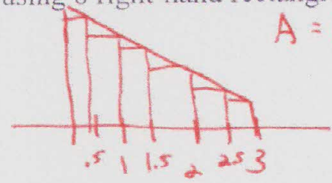
$v(t) = \int a(t)dt = \int -32\,dt$

$= -32t + C$

$v(t) = -32t + 56 = 0$

$t = \frac{56}{32} = 1.75$ sec

6. Find the area for the region bounded by $f(x) = 9 - x$ and the x-axis between $x = 0$ and $x = 3$ using 6 right-hand rectangles.



$A = (.5)(9 - \frac{1}{2}) + (.5)(9 - 1)$

$+ .5(9 - 1.5) + .5(9 - 2)$

$+ .5(9 - 2.5) + .5(9 - 3)$

$= 21.75$

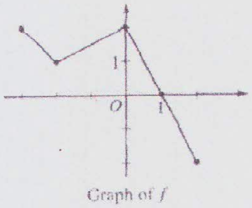
7. Evaluate: $\int \sqrt[5]{x^2}\,dx$ $= \int x^{2/5} dx$

$= \frac{5}{7}x^{7/5} + C$

8. Evaluate the integral: $\int (ax + b)dx$

$\frac{ax^2}{2} + bx + C$

9.



Graph of f

The graph of the piecewise linear function f is shown. If $g(x) = \int_{-2}^{x} f(t)dt$, which of the following values is greatest?

$\int_{-2}^{-2} f(t) = 0$

$\int_{-2}^{-3} f(t)dt = -\#$

− 0 + + bigger + - some(-)

A. g(−3) B. g(−2) C. g(0) [✓ D. g(1)] E. g(2)

10. Evaluate: $\int 3\sec^2 x dx$ $= 3\tan x + C$

11. Find the function, $y = f(x)$, if $f''(x) = x^2$, $f'(0) = 7$ and $f(0) = 2$.

$f'(x) = \int x^2 dx = \frac{x^3}{3} + C$

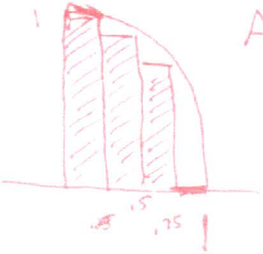
$f'(x) = \frac{x^3}{3} + 7$

$f(x) = \int\left(\frac{x^3}{3} + 7\right)dx$

$= \frac{x^4}{12} + 7x + C$

$\boxed{f(x) = \frac{x^4}{12} + 7x + 2}$

12. Find the lower sum for the region bounded by $f(x) = 1 - x^2$ and the x-axis between $x = 0$ and $x = 1$ using 4 rectangles.

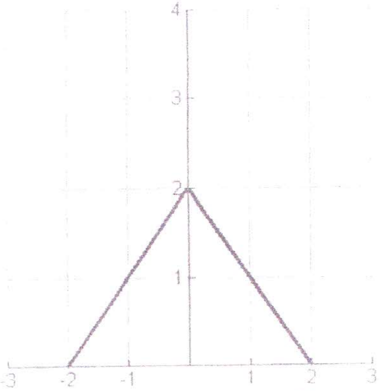


$A = (.25)\left[(1-.25^2) + (1-.5^2) + (1-.75)^2 + 0\right]$

$= .53125$

$= \frac{17}{32}$

13. Write the definite integral that represents the area under the curve for the following problem. Then evaluate the integral.



$\int_{-2}^{0} (x+2)\,dx + \int_{0}^{2} -x+2\,dx$

or

$\int_{-2}^{2} \left| -x+2 \right|$

14. Evaluate the integral: $\int \frac{\sin^3 \theta}{1-\cos^2 \theta} d\theta$.

$= \int \frac{\sin^3 x}{\sin^2 x} dx = \int \sin x\,dx = -\cos x + C$

$\boxed{= -\cos \theta + C}$

15. Evaluate: $\int \frac{\sec^3 \theta \tan \theta}{1+\tan^2 \theta} d\theta$. $= \int \frac{\sec^3 x \tan x}{\sec^2 x}$

$= \int \sec x \tan x = \sec x + C$

16. A ball is dropped from a height of 300 ft. Its velocity after t seconds is $v = -32t$ ft/sec.

$x(t) = \int v(t)$

$= \int -32t = \frac{-32t^2}{2} + C$

$= -16t^2 + C$

$300 = -16(0)^2 + C$

$C = 300$

A. How fast is the ball dropping after 4 seconds?

$v(4) = (-32)(4) = -128$ ft/sec

B. Determine the position function.

$x(t) = -16t^2 + 300$

C. How far has the ball dropped after 4 seconds?

$x(4) = 44$ ft; so if started at 300 and is now at 44, it must have fallen 256ft

D. How many seconds will it take for the ball to hit the ground?

$x(t) = 0 \quad -16t^2 + 300 = 0$

$t^2 = \frac{300}{16}$

$\boxed{t = 4.330 \text{ sec}}$

Given $\int_{4}^{0} f(x)dx = 5$ and $\int_{0}^{3} f(x)dx = 7$ find the following:

17. $\int_{4}^{3} f(x)dx$

$= 5 + 7$

$= 12$

18. $\int_{2}^{1} [f(x+2)]dx$ shifted left 2

$= \int_{0}^{3} f(x)\,dx$

$= 7$

19. $\int_{3}^{3} f(x)dx = 0$