

p.191 Key

3. $f(x) = 186.5$ is a constant function, so its derivative is 0, that is, $f'(x) = 0$.

5. $f(x) = x^3 - 4x + 6 \Rightarrow f'(x) = 3x^2 - 4(1) + 0 = 3x^2 - 4$

7. $f(t) = \frac{1}{4}(t^4 + 8) \Rightarrow f'(t) = \frac{1}{4}(t^4 + 8)' = \frac{1}{4}(4t^{4-1} + 0) = t^3$

9. $y = x^{-2/5} \Rightarrow y' = -\frac{2}{5}x^{(-2/5)-1} = -\frac{2}{5}x^{-7/5} = -\frac{2}{5x^{7/5}}$

11. $G(x) = \sqrt{x} - 2e^x = x^{1/2} - 2e^x \Rightarrow G'(x) = \frac{1}{2}x^{-1/2} - 2e^x = \frac{1}{2\sqrt{x}} - 2e^x$

13. $V(r) = \frac{4}{3}\pi r^3 \Rightarrow V'(r) = \frac{4}{3}\pi(3r^2) = 4\pi r^2$

15. $F(x) = (\frac{1}{2}x)^5 = (\frac{1}{2})^5 x^5 = \frac{1}{32}x^5 \Rightarrow F'(x) = \frac{1}{32}(5x^4) = \frac{5}{32}x^4$

17. $y = 4\pi^2 \Rightarrow y' = 0$ since $4\pi^2$ is a constant.

19. $y = \frac{x^2 + 4x + 3}{\sqrt{x}} = x^{3/2} + 4x^{1/2} + 3x^{-1/2} \Rightarrow$
 $y' = \frac{3}{2}x^{1/2} + 4(\frac{1}{2})x^{-1/2} + 3(-\frac{1}{2})x^{-3/2} = \frac{3}{2}\sqrt{x} + \frac{2}{\sqrt{x}} - \frac{3}{2x\sqrt{x}}$
 [note that $x^{3/2} = x^{2/2} \cdot x^{1/2} = x\sqrt{x}$]

21. $v = t^2 - \frac{1}{\sqrt[4]{t^3}} = t^2 - t^{-3/4} \Rightarrow v' = 2t - (-\frac{3}{4})t^{-7/4} = 2t + \frac{3}{4t^{7/4}} = 2t + \frac{3}{4t\sqrt[4]{t^3}}$

23. $z = \frac{A}{y^{10}} + Be^y = Ay^{-10} + Be^y \Rightarrow z' = -10Ay^{-11} + Be^y = -\frac{10A}{y^{11}} + Be^y$

26. $y = (1 + 2x)^2 = 1 + 4x + 4x^2 \Rightarrow y' = 4 + 8x$. At $(1, 9)$, $y' = 12$ and an equation of the tangent line is $y - 9 = 12(x - 1)$ or $y = 12x - 3$. The slope of the normal line is $-\frac{1}{12}$ (the negative reciprocal of 12) and an equation of the normal line is $y - 9 = -\frac{1}{12}(x - 1)$ or $y = -\frac{1}{12}x + \frac{109}{12}$.

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51. The slope of $y = x^2 - 5x + 4$ is given by $m = y' = 2x - 5$. The slope of $x - 3y = 5 \Leftrightarrow y = \frac{1}{3}x - \frac{5}{3}$ is $\frac{1}{3}$, so the desired normal line must have slope $\frac{1}{3}$, and hence, the tangent line to the parabola must have slope -3 . This occurs if $2x - 5 = -3 \Rightarrow 2x = 2 \Rightarrow x = 1$. When $x = 1, y = 1^2 - 5(1) + 4 = 0$, and an equation of the normal line is $y - 0 = \frac{1}{3}(x - 1)$ or $y = \frac{1}{3}x - \frac{1}{3}$.

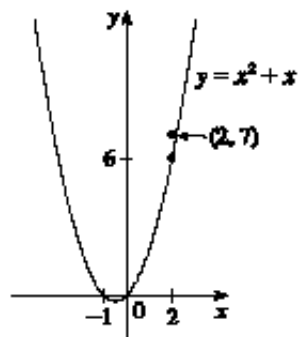
54. (a) If $y = x^2 + x$, then $y' = 2x + 1$. If the point at which a tangent meets the parabola is $(a, a^2 + a)$, then the slope of the tangent is $2a + 1$. But since it passes through $(2, -3)$, the slope must also be $\frac{\Delta y}{\Delta x} = \frac{a^2 + a + 3}{a - 2}$.

Therefore, $2a + 1 = \frac{a^2 + a + 3}{a - 2}$. Solving this equation for a we get $a^2 + a + 3 = 2a^2 - 3a - 2 \Leftrightarrow$

$a^2 - 4a - 5 = (a - 5)(a + 1) = 0 \Leftrightarrow a = 5$ or -1 . If $a = -1$, the point is $(-1, 0)$ and the slope is -1 , so the equation is $y - 0 = (-1)(x + 1)$ or $y = -x - 1$. If $a = 5$, the point is $(5, 30)$ and the slope is 11 , so the equation is $y - 30 = 11(x - 5)$ or $y = 11x - 25$.

(b) As in part (a), but using the point $(2, 7)$, we get the equation $2a + 1 = \frac{a^2 + a - 7}{a - 2}$

$\Rightarrow 2a^2 - 3a - 2 = a^2 + a - 7 \Leftrightarrow a^2 - 4a + 5 = 0$. The last equation has no real solution (discriminant $= -16 < 0$), so there is no line through the point $(2, 7)$ that is tangent to the parabola. The diagram shows that the point $(2, 7)$ is "inside" the parabola, but tangent lines to the parabola do not pass through points inside the parabola.



57. Let $P(x) = ax^2 + bx + c$. Then $P'(x) = 2ax + b$ and $P''(x) = 2a$. $P''(2) = 2 \Rightarrow 2a = 2 \Rightarrow a = 1$.

$$P'(2) = 3 \Rightarrow 2(1)(2) + b = 3 \Rightarrow 4 + b = 3 \Rightarrow b = -1.$$

$$P(2) = 5 \Rightarrow 1(2)^2 + (-1)(2) + c = 5 \Rightarrow 2 + c = 5 \Rightarrow c = 3. \text{ So } P(x) = x^2 - x + 3.$$

65. *Solution 1:* Let $f(x) = x^{1000}$. Then, by the definition of a derivative, $f'(1) = \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1} \frac{x^{1000} - 1}{x - 1}$.

But this is just the limit we want to find, and we know (from the Power Rule) that $f'(x) = 1000x^{999}$, so

$$f'(1) = 1000(1)^{999} = 1000. \text{ So } \lim_{x \rightarrow 1} \frac{x^{1000} - 1}{x - 1} = 1000.$$

Solution 2: Note that $(x^{1000} - 1) = (x - 1)(x^{999} + x^{998} + x^{997} + \cdots + x^2 + x + 1)$. So

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{x^{1000} - 1}{x - 1} &= \lim_{x \rightarrow 1} \frac{(x - 1)(x^{999} + x^{998} + x^{997} + \cdots + x^2 + x + 1)}{x - 1} = \lim_{x \rightarrow 1} (x^{999} + x^{998} + x^{997} + \cdots + x^2 + x + 1) \\ &= \underbrace{1 + 1 + 1 + \cdots + 1 + 1 + 1}_{1000 \text{ ones}} = 1000, \text{ as above.} \end{aligned}$$