

p.198 Key

3. By the Product Rule, $f(x) = x^2 e^x \Rightarrow f'(x) = x^2 \frac{d}{dx}(e^x) + e^x \frac{d}{dx}(x^2) = x^2 e^x + e^x(2x) = x e^x(x + 2)$.

6. By the Quotient Rule, $y = \frac{e^x}{1+x} \Rightarrow y' = \frac{(1+x)e^x - e^x(1)}{(1+x)^2} = \frac{e^x + x e^x - e^x}{(1+x)^2} = \frac{x e^x}{(1+x)^2}$.

The notations $\xRightarrow{\text{PR}}$ and $\xRightarrow{\text{QR}}$ indicate the use of the Product and Quotient Rules, respectively.

9. $F(y) = \left(\frac{1}{y^2} - \frac{3}{y^4}\right)(y + 5y^3) = (y^{-2} - 3y^{-4})(y + 5y^3) \xRightarrow{\text{PR}}$

$$\begin{aligned} F'(y) &= (y^{-2} - 3y^{-4})(1 + 15y^2) + (y + 5y^3)(-2y^{-3} + 12y^{-5}) \\ &= (y^{-2} + 15 - 3y^{-4} - 45y^{-2}) + (-2y^{-2} + 12y^{-4} - 10 + 60y^{-2}) \\ &= 5 + 14y^{-2} + 9y^{-4} \text{ or } 5 + 14/y^2 + 9/y^4 \end{aligned}$$

12. $y = \frac{t^3 + t}{t^4 - 2} \xRightarrow{\text{QR}} y' = \frac{(t^4 - 2)(3t^2 + 1) - (t^3 + t)(4t^3)}{(t^4 - 2)^2} = \frac{(3t^6 + t^4 - 6t^2 - 2) - (4t^6 + 4t^4)}{(t^4 - 2)^2}$
 $= \frac{-t^6 - 3t^4 - 6t^2 - 2}{(t^4 - 2)^2} = -\frac{t^6 + 3t^4 + 6t^2 + 2}{(t^4 - 2)^2}$

15. $y = \frac{v^3 - 2v\sqrt{v}}{v} = v^2 - 2\sqrt{v} = v^2 - 2v^{1/2} \Rightarrow y' = 2v - 2\left(\frac{1}{2}\right)v^{-1/2} = 2v - v^{-1/2}$.

We can change the form of the answer as follows: $2v - v^{-1/2} = 2v - \frac{1}{\sqrt{v}} = \frac{2v\sqrt{v} - 1}{\sqrt{v}} = \frac{2v^{3/2} - 1}{\sqrt{v}}$

18. $f(x) = \frac{1 - xe^x}{x + e^x} \xRightarrow{\text{QR}} f'(x) = \frac{(x + e^x)(-xe^x)' - (1 - xe^x)(1 + e^x)}{(x + e^x)^2}$

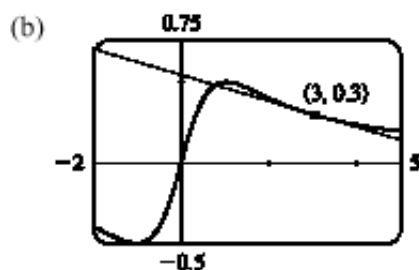
$$\begin{aligned} &\xRightarrow{\text{PR}} f'(x) = \frac{(x + e^x)[-(xe^x + e^x \cdot 1)] - (1 + e^x - xe^x - xe^{2x})}{(x + e^x)^2} \\ &= \frac{-x^2 e^x - xe^x - xe^{2x} - e^{2x} - 1 - e^x + xe^x + xe^{2x}}{(x + e^x)^2} = \frac{-x^2 e^x - e^{2x} - e^x - 1}{(x + e^x)^2} \end{aligned}$$

21. $y = 2xe^x \Rightarrow y' = 2(x \cdot e^x + e^x \cdot 1) = 2e^x((x + 1))$. At $(0, 0)$, $y' = 2e^0(0 + 1) = 2 \cdot 1 \cdot 1 = 2$, and an equation of the tangent line is $y - 0 = 2(x - 0)$, or $y = 2x$. The slope of the normal line is $-\frac{1}{2}$, so an equation of the normal line is $y - 0 = -\frac{1}{2}(x - 0)$, or $y = -\frac{1}{2}x$.

24. (a) $y = f(x) = \frac{x}{1+x^2} \Rightarrow$

$$f'(x) = \frac{(1+x^2)1 - x(2x)}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}. \text{ So the slope of the}$$

tangent line at the point $(3, 0.3)$ is $f'(3) = \frac{-8}{100}$ and its equation is $y - 0.3 = -0.08(x - 3)$ or $y = -0.08x + 0.54$.



31. We are given that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$, and $g'(5) = 2$.

(a) $(fg)'(5) = f(5)g'(5) + g(5)f'(5) = (1)(2) + (-3)(6) = 2 - 18 = -16$

(b) $\left(\frac{f}{g}\right)'(5) = \frac{g(5)f'(5) - f(5)g'(5)}{[g(5)]^2} = \frac{(-3)(6) - (1)(2)}{(-3)^2} = -\frac{20}{9}$

(c) $\left(\frac{g}{f}\right)'(5) = \frac{f(5)g'(5) - g(5)f'(5)}{[f(5)]^2} = \frac{(1)(2) - (-3)(6)}{(1)^2} = 20$

35. (a) From the graphs of f and g , we obtain the following values: $f(1) = 2$ since the point $(1, 2)$ is on the graph of f ;

$g(1) = 1$ since the point $(1, 1)$ is on the graph of g ; $f'(1) = 2$ since the slope of the line segment between $(0, 0)$ and $(2, 4)$

is $\frac{4-0}{2-0} = 2$; $g'(1) = -1$ since the slope of the line segment between $(-2, 4)$ and $(2, 0)$ is $\frac{0-4}{2-(-2)} = -1$.

Now $u(x) = f(x)g(x)$, so $u'(1) = f(1)g'(1) + g(1)f'(1) = 2 \cdot (-1) + 1 \cdot 2 = 0$.

(b) $v(x) = f(x)/g(x)$, so $v'(5) = \frac{g(5)f'(5) - f(5)g'(5)}{[g(5)]^2} = \frac{2(-\frac{1}{3}) - 3 \cdot \frac{2}{3}}{2^2} = \frac{-\frac{8}{3}}{4} = -\frac{2}{3}$

36. (a) $P(x) = F(x)G(x)$, so $P'(2) = F(2)G'(2) + G(2)F'(2) = 3 \cdot \frac{2}{4} + 2 \cdot 0 = \frac{3}{2}$.

(b) $Q(x) = F(x)/G(x)$, so $Q'(7) = \frac{G(7)F'(7) - F(7)G'(7)}{[G(7)]^2} = \frac{1 \cdot \frac{1}{4} - 5 \cdot (-\frac{2}{3})}{1^2} = \frac{1}{4} + \frac{10}{3} = \frac{43}{12}$

$$38. (a) y = x^2 f(x) \Rightarrow y' = x^2 f'(x) + f(x)(2x)$$

$$(b) y = \frac{f(x)}{x^2} \Rightarrow y' = \frac{x^2 f'(x) - f(x)(2x)}{(x^2)^2} = \frac{xf'(x) - 2f(x)}{x^3}$$

$$(c) y = \frac{x^2}{f(x)} \Rightarrow y' = \frac{f(x)(2x) - x^2 f'(x)}{[f(x)]^2}$$

$$(d) y = \frac{1 + xf(x)}{\sqrt{x}} \Rightarrow$$

$$y' = \frac{\sqrt{x}[xf'(x) + f(x)] - [1 + xf(x)] \frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}$$

$$= \frac{x^{3/2} f'(x) + x^{1/2} f(x) - \frac{1}{2} x^{-1/2} - \frac{1}{2} x^{1/2} f(x)}{x} \cdot \frac{2x^{1/2}}{2x^{1/2}} = \frac{xf(x) + 2x^2 f'(x) - 1}{2x^{3/2}}$$