

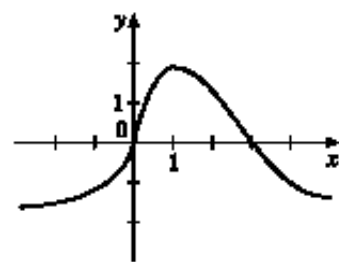
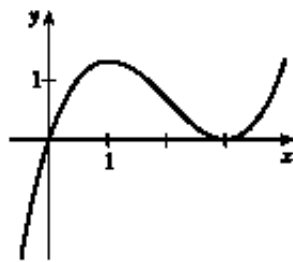
P.153 Key

3. $g'(0)$ is the only negative value. The slope at $x = 4$ is smaller than the slope at $x = 2$ and both are smaller than the slope at $x = -2$. Thus, $g'(0) < 0 < g'(4) < g'(2) < g'(-2)$.

4. (a) Since $g(5) = -3$, the point $(5, -3)$ is on the graph of g . Since $g'(5) = 4$, the slope of the tangent line at $x = 5$ is 4. Using the point-slope form of a line gives us $y - (-3) = 4(x - 5)$, or $y = 4x - 23$.

(b) Since $(4, 3)$ is on $y = f(x)$, $f(4) = 3$. The slope of the tangent line between $(0, 2)$ and $(4, 3)$ is $\frac{1}{4}$, so $f'(4) = \frac{1}{4}$.

5. We begin by drawing a curve through the origin with a slope of 3 to satisfy $f(0) = 0$ and $f'(0) = 3$. Since $f'(1) = 0$, we will round off our figure so that there is a horizontal tangent directly over $x = 1$. Lastly, we make sure that the curve has a slope of -1 as we pass over $x = 2$. Two of the many possibilities are shown.



7. Using Definition 2 with $f(x) = 3x^2 - 5x$ and the point $(2, 2)$, we have

$$\begin{aligned} f'(2) &= \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{[3(2+h)^2 - 5(2+h)] - 2}{h} \\ &= \lim_{h \rightarrow 0} \frac{(12 + 12h + 3h^2 - 10 - 5h) - 2}{h} = \lim_{h \rightarrow 0} \frac{3h^2 + 7h}{h} = \lim_{h \rightarrow 0} (3h + 7) = 7 \end{aligned}$$

So an equation of the tangent line at $(2, 2)$ is $y - 2 = 7(x - 2)$ or $y = 7x - 12$.

15. Use Definition 2 with $f(t) = (2t + 1)/(t + 3)$.

$$\begin{aligned} f'(a) &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{\frac{2(a+h) + 1}{(a+h) + 3} - \frac{2a + 1}{a + 3}}{h} = \lim_{h \rightarrow 0} \frac{(2a + 2h + 1)(a + 3) - (2a + 1)(a + h + 3)}{h(a + h + 3)(a + 3)} \\ &= \lim_{h \rightarrow 0} \frac{(2a^2 + 6a + 2ah + 6h + a + 3) - (2a^2 + 2ah + 6a + a + h + 3)}{h(a + h + 3)(a + 3)} \\ &= \lim_{h \rightarrow 0} \frac{5h}{h(a + h + 3)(a + 3)} = \lim_{h \rightarrow 0} \frac{5}{(a + h + 3)(a + 3)} = \frac{5}{(a + 3)^2} \end{aligned}$$

Note that the answers to Exercises 19–24 are not unique.

19. By Definition 2, $\lim_{h \rightarrow 0} \frac{(1+h)^{10} - 1}{h} = f'(1)$, where $f(x) = x^{10}$ and $a = 1$.

Or: By Definition 2, $\lim_{h \rightarrow 0} \frac{(1+h)^{10} - 1}{h} = f'(0)$, where $f(x) = (1+x)^{10}$ and $a = 0$.

P.153 Key

Note that the answers to Exercises 19–24 are not unique.

20. By Definition 2, $\lim_{h \rightarrow 0} \frac{\sqrt[4]{16+h} - 2}{h} = f'(16)$, where $f(x) = \sqrt[4]{x}$ and $a = 16$.

Or: By Definition 2, $\lim_{h \rightarrow 0} \frac{\sqrt[4]{16+h} - 2}{h} = f'(0)$, where $f(x) = \sqrt[4]{16+x}$ and $a = 0$.

Note that the answers to Exercises 19–24 are not unique.

21. By Equation 3, $\lim_{x \rightarrow 5} \frac{2^x - 32}{x - 5} = f'(5)$, where $f(x) = 2^x$ and $a = 5$.

Note that the answers to Exercises 19–24 are not unique.

22. By Equation 3, $\lim_{x \rightarrow \pi/4} \frac{\tan x - 1}{x - \pi/4} = f'(\pi/4)$, where $f(x) = \tan x$ and $a = \pi/4$.

Note that the answers to Exercises 19–24 are not unique.

23. By Definition 2, $\lim_{h \rightarrow 0} \frac{\cos(\pi + h) + 1}{h} = f'(\pi)$, where $f(x) = \cos x$ and $a = \pi$.

Or: By Definition 2, $\lim_{h \rightarrow 0} \frac{\cos(\pi + h) + 1}{h} = f'(0)$, where $f(x) = \cos(\pi + x)$ and $a = 0$.

Note that the answers to Exercises 19–24 are not unique.

24. By Equation 3, $\lim_{t \rightarrow 1} \frac{t^4 + t - 2}{t - 1} = f'(1)$, where $f(t) = t^4 + t$ and $a = 1$.

30. (a) $f'(8)$ is the rate of change of the quantity of coffee sold with respect to the price per pound when the price is \$8 per pound. The units for $f'(8)$ are pounds/(dollars/pound).

(b) $f'(8)$ is negative since the quantity of coffee sold will decrease as the price charged for it increases. People are generally less willing to buy a product when its price increases.

31. $T'(10)$ is the rate at which the temperature is changing at 10:00 A.M. To estimate the value of $T'(10)$, we will average the difference quotients obtained using the times $t = 8$ and $t = 12$. Let $A = \frac{T(8) - T(10)}{8 - 10} = \frac{72 - 81}{-2} = 4.5$ and

$$B = \frac{T(12) - T(10)}{12 - 10} = \frac{88 - 81}{2} = 3.5. \text{ Then } T'(10) = \lim_{t \rightarrow 10} \frac{T(t) - T(10)}{t - 10} \approx \frac{A + B}{2} = \frac{4.5 + 3.5}{2} = 4 \text{ }^\circ\text{F/h.}$$