

1. Let $u = g(x) = 4x$ and $y = f(u) = \sin u$. Then $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = (\cos u)(4) = 4 \cos 4x$.

2. Let $u = g(x) = 4 + 3x$ and $y = f(u) = \sqrt{u} = u^{1/2}$. Then $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = \frac{1}{2}u^{-1/2}(3) = \frac{3}{2\sqrt{u}} = \frac{3}{2\sqrt{4+3x}}$.

3. Let $u = g(x) = 1 - x^2$ and $y = f(u) = u^{10}$. Then $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = (10u^9)(-2x) = -20x(1 - x^2)^9$.

4. Let $u = g(x) = \sin x$ and $y = f(u) = \tan u$. Then $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = (\sec^2 u)(\cos x) = \sec^2(\sin x) \cdot \cos x$,
or equivalently, $[\sec(\sin x)]^2 \cos x$.

5. Let $u = g(x) = \sqrt{x}$ and $y = f(u) = e^u$. Then $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = (e^u)\left(\frac{1}{2}x^{-1/2}\right) = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$.

6. Let $u = g(x) = e^x$ and $y = f(u) = \sin u$. Then $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = (\cos u)(e^x) = e^x \cos e^x$.

7. $F(x) = \sqrt[4]{1+2x+x^3} = (1+2x+x^3)^{1/4} \Rightarrow$

$$F'(x) = \frac{1}{4}(1+2x+x^3)^{-3/4} \cdot \frac{d}{dx}(1+2x+x^3) = \frac{1}{4(1+2x+x^3)^{3/4}} \cdot (2+3x^2)$$

$$= \frac{2+3x^2}{4(1+2x+x^3)^{3/4}} = \frac{2+3x^2}{4\sqrt[4]{(1+2x+x^3)^3}}$$

8. $F(x) = (x^2 - x + 1)^3 \Rightarrow F'(x) = 3(x^2 - x + 1)^2(2x - 1)$

9. $g(t) = \frac{1}{(t^4 + 1)^3} = (t^4 + 1)^{-3} \Rightarrow g'(t) = -3(t^4 + 1)^{-4}(4t^3) = -12t^3(t^4 + 1)^{-4} = \frac{-12t^3}{(t^4 + 1)^4}$

10. $f(t) = \sqrt[3]{1 + \tan t} = (1 + \tan t)^{1/3} \Rightarrow f'(t) = \frac{1}{3}(1 + \tan t)^{-2/3} \sec^2 t = \frac{\sec^2 t}{3\sqrt[3]{(1 + \tan t)^2}}$

41. $F(x) = f(g(x)) \Rightarrow F'(x) = f'(g(x)) \cdot g'(x)$, so $F'(5) = f'(g(5)) \cdot g'(5) = f'(-2) \cdot 6 = 4 \cdot 6 = 24$

42. $h(x) = \sqrt{4 + 3f(x)} \Rightarrow h'(x) = \frac{1}{2}(4 + 3f(x))^{-1/2} \cdot 3f'(x)$, so
 $h'(1) = \frac{1}{2}(4 + 3f(1))^{-1/2} \cdot 3f'(1) = \frac{1}{2}(4 + 3 \cdot 7)^{-1/2} \cdot 3 \cdot 4 = \frac{6}{\sqrt{25}} = \frac{6}{5}$
43. (a) $h(x) = f(g(x)) \Rightarrow h'(x) = f'(g(x)) \cdot g'(x)$, so $h'(1) = f'(g(1)) \cdot g'(1) = f'(2) \cdot 6 = 5 \cdot 6 = 30$.
 (b) $H(x) = g(f(x)) \Rightarrow H'(x) = g'(f(x)) \cdot f'(x)$, so $H'(1) = g'(f(1)) \cdot f'(1) = g'(3) \cdot 4 = 9 \cdot 4 = 36$.
44. (a) $F(x) = f(f(x)) \Rightarrow F'(x) = f'(f(x)) \cdot f'(x)$, so $F'(2) = f'(f(2)) \cdot f'(2) = f'(1) \cdot 5 = 4 \cdot 5 = 20$.
 (b) $G(x) = g(g(x)) \Rightarrow G'(x) = g'(g(x)) \cdot g'(x)$, so $G'(3) = g'(g(3)) \cdot g'(3) = g'(2) \cdot 9 = 7 \cdot 9 = 63$.
45. (a) $u(x) = f(g(x)) \Rightarrow u'(x) = f'(g(x))g'(x)$. So $u'(1) = f'(g(1))g'(1) = f'(3)g'(1)$. To find $f'(3)$, note that f is linear from $(2, 4)$ to $(6, 3)$, so its slope is $\frac{3-4}{6-2} = -\frac{1}{4}$. To find $g'(1)$, note that g is linear from $(0, 6)$ to $(2, 0)$, so its slope is $\frac{0-6}{2-0} = -3$. Thus, $f'(3)g'(1) = (-\frac{1}{4})(-3) = \frac{3}{4}$.
- (b) $v(x) = g(f(x)) \Rightarrow v'(x) = g'(f(x))f'(x)$. So $v'(1) = g'(f(1))f'(1) = g'(2)f'(1)$, which does not exist since $g'(2)$ does not exist.
- (c) $w(x) = g(g(x)) \Rightarrow w'(x) = g'(g(x))g'(x)$. So $w'(1) = g'(g(1))g'(1) = g'(3)g'(1)$. To find $g'(3)$, note that g is linear from $(2, 0)$ to $(5, 2)$, so its slope is $\frac{2-0}{5-2} = \frac{2}{3}$. Thus, $g'(3)g'(1) = (\frac{2}{3})(-3) = -2$.