

$$3. f(\theta) = \ln(\cos \theta) \Rightarrow f'(\theta) = \frac{1}{\cos \theta} \frac{d}{d\theta} (\cos \theta) = \frac{-\sin \theta}{\cos \theta} = -\tan \theta$$

$$6. f(x) = \ln \sqrt[5]{x} = \ln x^{1/5} = \frac{1}{5} \ln x \Rightarrow f'(x) = \frac{1}{5} \cdot \frac{1}{x} = \frac{1}{5x}$$

$$9. f(x) = \sin x \ln(5x) \Rightarrow f'(x) = \sin x \cdot \frac{1}{5x} \cdot \frac{d}{dx} (5x) + \ln(5x) \cdot \cos x \\ = \frac{\sin x \cdot 5}{5x} + \cos x \ln(5x) = \frac{\sin x}{x} + \cos x \ln(5x)$$

$$12. F(y) = y \ln(1 + e^y) \Rightarrow F'(y) = y \cdot \frac{1}{1 + e^y} \cdot e^y + \ln(1 + e^y) \cdot 1 = \frac{ye^y}{1 + e^y} + \ln(1 + e^y)$$

$$15. y = \ln |2 - x - 5x^2| \Rightarrow y' = \frac{1}{2 - x - 5x^2} \cdot (-1 - 10x) = \frac{-10x - 1}{2 - x - 5x^2} \text{ or } \frac{10x + 1}{5x^2 + x - 2}$$

$$18. y = [\ln(1 + e^x)]^2 \Rightarrow y' = 2[\ln(1 + e^x)] \cdot \frac{1}{1 + e^x} \cdot e^x = \frac{2e^x \ln(1 + e^x)}{1 + e^x}$$

$$21. f(x) = \frac{x}{1 - \ln(x-1)} \Rightarrow$$

$$f'(x) = \frac{[1 - \ln(x-1)] \cdot 1 - x \cdot \frac{-1}{x-1}}{[1 - \ln(x-1)]^2} = \frac{(x-1)[1 - \ln(x-1)] + x}{[1 - \ln(x-1)]^2} = \frac{x-1 - (x-1)\ln(x-1) + x}{(x-1)[1 - \ln(x-1)]^2} \\ = \frac{2x-1 - (x-1)\ln(x-1)}{(x-1)[1 - \ln(x-1)]^2}$$

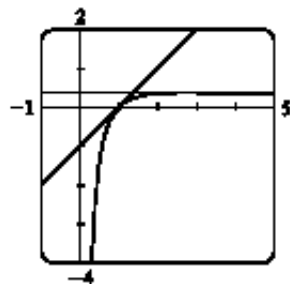
$$\text{Dom}(f) = \{x \mid x-1 > 0 \text{ and } 1 - \ln(x-1) \neq 0\} = \{x \mid x > 1 \text{ and } \ln(x-1) \neq 1\} \\ = \{x \mid x > 1 \text{ and } x-1 \neq e^1\} = \{x \mid x > 1 \text{ and } x \neq 1+e\} = (1, 1+e) \cup (1+e, \infty)$$

$$24. y = \frac{\ln x}{x} \Rightarrow y' = \frac{x(1/x) - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$$

$$y'(1) = \frac{1-0}{1^2} = 1 \text{ and } y'(e) = \frac{1-1}{e^2} = 0 \Rightarrow \text{equations of tangent}$$

$$\text{lines are } y - 0 = 1(x - 1) \text{ or } y = x - 1 \text{ and } y - 1/e = 0(x - e)$$

$$\text{or } y = 1/e.$$



$$\begin{aligned}
 27. \quad y &= (2x+1)^5(x^4-3)^6 \Rightarrow \ln y = \ln((2x+1)^5(x^4-3)^6) \Rightarrow \ln y = 5\ln(2x+1) + 6\ln(x^4-3) \Rightarrow \\
 \frac{1}{y} y' &= 5 \cdot \frac{1}{2x+1} \cdot 2 + 6 \cdot \frac{1}{x^4-3} \cdot 4x^3 \Rightarrow \\
 y' &= y \left(\frac{10}{2x+1} + \frac{24x^3}{x^4-3} \right) = (2x+1)^5(x^4-3)^6 \left(\frac{10}{2x+1} + \frac{24x^3}{x^4-3} \right).
 \end{aligned}$$

[The answer could be simplified to $y' = 2(2x+1)^4(x^4-3)^5(29x^4+12x^3-15)$, but this is unnecessary.]

$$\begin{aligned}
 32. \quad y &= x^{\cos x} \Rightarrow \ln y = \ln x^{\cos x} \Rightarrow \ln y = \cos x \ln x \Rightarrow \frac{1}{y} y' = \cos x \cdot \frac{1}{x} + \ln x \cdot (-\sin x) \Rightarrow \\
 y' &= y \left(\frac{\cos x}{x} - \ln x \sin x \right) \Rightarrow y' = x^{\cos x} \left(\frac{\cos x}{x} - \ln x \sin x \right)
 \end{aligned}$$

$$\begin{aligned}
 38. \quad x^y &= y^x \Rightarrow y \ln x = x \ln y \Rightarrow y \cdot \frac{1}{x} + (\ln x) \cdot y' = x \cdot \frac{1}{y} \cdot y' + \ln y \Rightarrow y' \ln x - \frac{x}{y} y' = \ln y - \frac{y}{x} \Rightarrow \\
 y' &= \frac{\ln y - y/x}{\ln x - x/y}
 \end{aligned}$$

$$\begin{aligned}
 39. \quad f(x) &= \ln(x-1) \Rightarrow f'(x) = 1/(x-1) = (x-1)^{-1} \Rightarrow f''(x) = -(x-1)^{-2} \Rightarrow f'''(x) = 2(x-1)^{-3} \Rightarrow \\
 f^{(4)}(x) &= -2 \cdot 3(x-1)^{-4} \Rightarrow \dots \Rightarrow f^{(n)}(x) = (-1)^{n-1} \cdot 2 \cdot 3 \cdot 4 \cdot \dots \cdot (n-1)(x-1)^{-n} = (-1)^{n-1} \frac{(n-1)!}{(x-1)^n}
 \end{aligned}$$