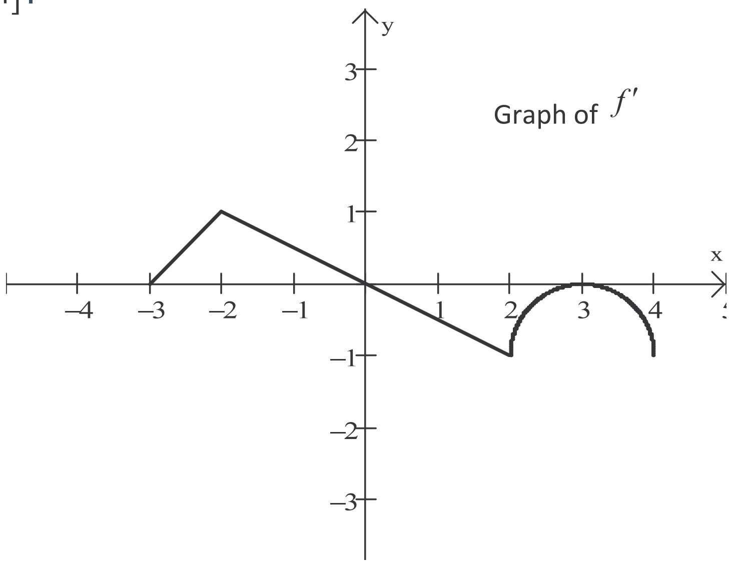
Name________________________________ **#1 Justify Your Answer**

The graph of the continuous function f' consists of two line segments and a half circle as shown in the graph below. The point $(2,3)$ is on the graph of the function f whose domain is the closed interval $[-3,4]$.



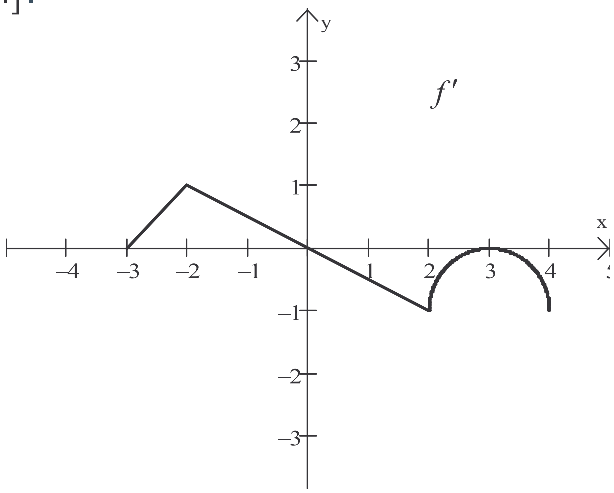
(a) What are the x-coordinates of any relative maxima of f? Justify your answer.

(b) What is the absolute maximum value of f? Justify your answer.

(c) What is the absolute minimum value of f? Justify your answer.

(d) What are the x-coordinates of any points of inflection of f? Justify your answer.

The graph of the continuous function f' consists of two line segments and a half circle as shown in the graph below. The point $(2,3)$ is on the graph of the function f whose domain is the closed interval $[-3,4]$.



(a) What are the *x*-coordinates of any relative maxima of f? Justify your answer.

	Explanation given	Awarded points on AP exam?
1	f has a rel max at $x=0$ because f' changes from positive to negative there	
2	f has a rel max at $x=0$ because f changes from increasing to decreasing there.	
3	f has a rel max at $x=0$ because $f'(0)=0$.	

(b) What is the absolute maximum value of f? Justify your answer.

	Explanation given	Awarded points on AP exam?
1	f has an abs max at $x=0$ because $f'>0$ for $-3<x<0$ and $f'<0$ for $0<x<3$ and $3<x<4$. $f(0)=f(2)+\int_2^0 f'(x)\,dx=3-\left(-\frac{1}{2}\cdot 2\cdot 1\right)=4$.	
2	f has an abs max at $x=0$ because f' changes from positive to negative there. $f(0)=f(2)+\int_2^0 f'(x)\,dx=4$.	
3	f has a rel max at $x=0$ because f' changes from positive to negative there. This is the only rel max so it is an abs max. $f(0)=f(2)+\int_2^0 f'(x)\,dx=4$.	
4	f has a rel max at $x=0$ because f' changes from positive to negative there. This is the only critical number so it is an abs max. $f(0)=f(2)+\int_2^0 f'(x)\,dx=4$.	

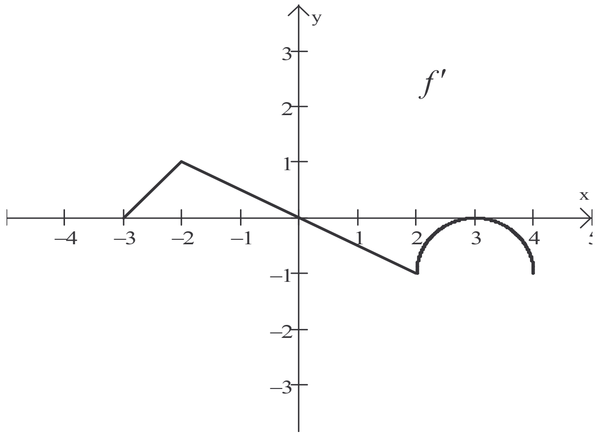
<table>
<tr><td>5</td><td>
<table>
<tr><th>x</th><th>$f(x)$</th></tr>
<tr><td>-3</td><td>$f(2)+\int_2^{-3} f'(x)dx=3-\left(\frac{1}{2}\cdot 1\cdot 1\right)=2.5$</td></tr>
<tr><td>0</td><td>$f(2)+\int_2^{0} f'(x)dx=3-\left(-\frac{1}{2}\cdot 2\cdot 1\right)=4$</td></tr>
<tr><td>3</td><td>$f(2)+\int_2^{3} f'(x)dx=3-\left(1-\frac{\pi}{4}\right)=2+\frac{\pi}{4}\approx 2.8$</td></tr>
<tr><td>4</td><td>$f(2)+\int_2^{4} f'(x)dx=3-\left(2-\frac{\pi}{2}\right)=1+\frac{\pi}{2}\approx 2.6$</td></tr>
</table>

$\therefore$ the abs max value of f is 4.
</td><td></td></tr>
<tr><td>6</td><td>
The only rel max of f is at $x=0$ because only there does f' change from positive to negative.

<table>
<tr><th>x</th><th>$f(x)$</th></tr>
<tr><td>-3</td><td>$f(2)+\int_2^{-3} f'(x)dx=3-\left(\frac{1}{2}\cdot 1\cdot 1\right)=2.5$</td></tr>
<tr><td>0</td><td>$f(2)+\int_2^{0} f'(x)dx=3-\left(-\frac{1}{2}\cdot 2\cdot 1\right)=4$</td></tr>
<tr><td>4</td><td>$f(2)+\int_2^{4} f'(x)dx=3-\left(2-\frac{\pi}{2}\right)=1+\frac{\pi}{2}\approx 2.6$</td></tr>
</table>

$\therefore$ the abs max value of f is 4.
</td><td></td></tr>
<tr><td>7</td><td>
f has an abs max at $x=0$ because $f'>0$ for $x<0$ and $f'<0$ for $x>0$. $f(0)=f(2)+\int_2^{0} f'(x)\,dx=3-\left(-\frac{1}{2}\cdot 2\cdot 1\right)=4$.
</td><td></td></tr>
</table>

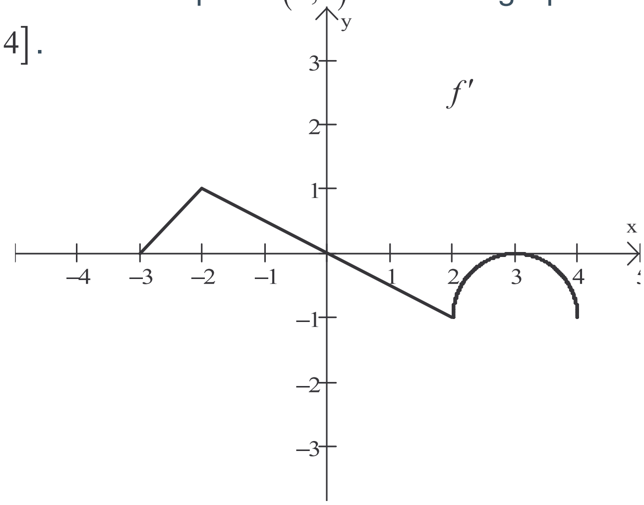
The graph of the continuous function f' consists of two line segments and a half circle as shown in the graph below. The point $(2,3)$ is on the graph of the function f whose domain is the closed interval $[-3,4]$.



(c) What is the absolute minimum value of f? Justify your answer.

	Explanation given	Awarded points on AP exam?
1	$f(-3)=f(2)+\int_2^{-3} f'(x)\,dx = 3-\left(\frac{1}{2}\cdot 1\cdot 1\right)=2.5$. . $f(4)=f(2)+\int_2^{4} f'(x)\,dx = 3-\left(2-\frac{\pi}{2}\right)=1+\frac{\pi}{2}\approx 2.6$ f has no local min since f' does not change from negative to positive. Therefore the absolute minimum value of f is 2.5	
2	x | $f(x)$ -3 | $f(2)+\int_2^{-3} f'(x)dx=3-\left(\frac{1}{2}\cdot1\cdot1\right)=2.5$ 0 | $f(2)+\int_2^{0} f'(x)dx=3-\frac{1}{2}\cdot2\cdot1=4$ 3 | $f(2)+\int_2^{3} f'(x)dx=3-\left(1-\frac{\pi}{4}\right)=2+\frac{\pi}{4}\approx2.8$ 4 | $f(2)+\int_2^{4} f'(x)dx=3-\left(2-\frac{\pi}{2}\right)=1+\frac{\pi}{2}\approx2.6$. Therefore the absolute minimum value of f is 2.5.	

The graph of the continuous function f' consists of two line segments and a half circle as shown in the graph below. The point $(2,3)$ is on the graph of the function f whose domain is the closed interval $[-3,4]$.

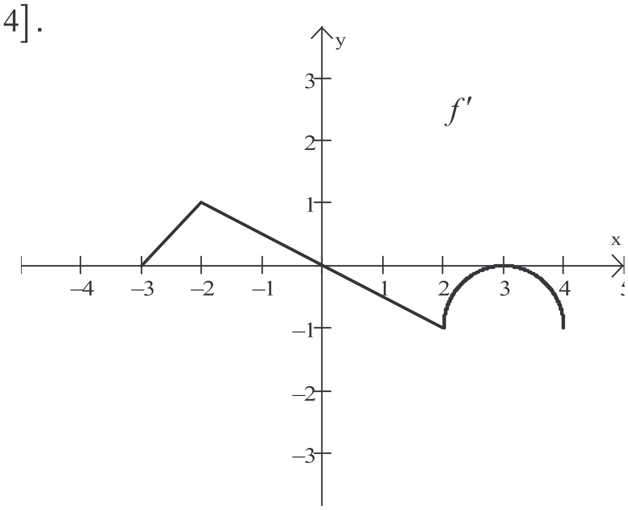


(d) What are the *x*-coordinates of any points of inflection of f? Justify your answer.

	Explanation given	Awarded points on AP exam?
1	$x=-2, 2, 3$ because f' has extrema at these points and so f has POI there.	
2	$x=-2, 2, 3$ because f' changes from increasing to decreasing at these points and so f has POI there.	
3	$x=-2, 2, 3$ because f' changes increasing/decreasing behavior at these points and so f has POI there.	
4	$x=-2, 2, 3$ because f' changes from increasing to decreasing or vice versa at these points and so f has POI there.	
5	f' is increasing on $(-3,-2)$ and $(2,3)$, and f' is decreasing on $(-2,2)$ and $(3,4)$. Therefore f has POI at $x=-2, 2, 3$.	
6	f has POI where f' changes increasing/decreasing behavior. f has POI at $x=-2, 2, 3$.	
7	f has POI where f' changes increasing/decreasing behavior. Therefore f has POI at $x=-2, 2, 3$.	
8	$x=-2, 2, 3$ because the slope of f' changes sign at these points and so f has POI there.	
9	$x=-2, 2, 3$ because f'' changes sign at these points and so f has POI there.	
10	$x=-2, 2, 3$ because the slope of f' changes from increasing to decreasing or from decreasing to increasing at these points and so f has POI there.	

KEY

The graph of the continuous function f' consists of two line segments and a half circle as shown in the graph below. The point $(2,3)$ is on the graph of the function f whose domain is the closed interval $[-3,4]$.



(a) What are the *x*-coordinates of any relative maxima of f? Justify your answer.

	Explanation given	Awarded points on AP exam?
1	f has a rel max at $x=0$ because f' changes from positive to negative there	*fine solution*
2	f has a rel max at $x=0$ because f changes from increasing to decreasing there.	*No. This merely describes what it means to have a maximum. The justification should clearly reference the "evidence", f'.*
3	f has a rel max at $x=0$ because $f'(0)=0$.	*No. This is an insufficient justification.*

(b) What is the absolute maximum value of f? Justify your answer.

	Explanation given	Awarded points on AP exam?
1	f has an abs max at $x=0$ because $f'>0$ for $-3<x<0$ and $f'<0$ for $0<x<3$ and $3<x<4$. $f(0)=f(2)+\int_2^0 f'(x)\,dx = 3-\left(-\frac{1}{2}\cdot 2\cdot 1\right)=4$.	*fine solution*
2	f has an abs max at $x=0$ because f' changes from positive to negative there. $f(0)=f(2)+\int_2^0 f'(x)\,dx=4$.	*No. This is a local argument.*

<table>
<tr><td>3</td><td>f has a rel max at $x=0$ because f' changes from positive to negative there.
This is the only rel max so it is an abs max. $f(0)=f(2)+\int_2^0 f'(x)\,dx=4$.</td><td>No. Having only one relative max is not sufficient to make it an absolute max.</td></tr>
<tr><td>4</td><td>f has a rel max at $x=0$ because f' changes from positive to negative there.
This is the only critical number so it is an abs max.
$f(0)=f(2)+\int_2^0 f'(x)\,dx=4$.</td><td>No. Ordinarily this is a good argument, but unfortunately, there is another critical point.</td></tr>
<tr><td>5</td><td>

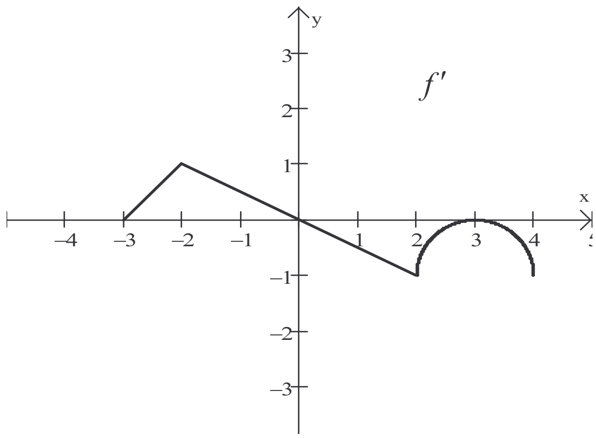
x	$f(x)$
-3	$f(2)+\int_2^{-3} f'(x)dx=3-\left(\frac{1}{2}\cdot 1\cdot 1\right)=2.5$
0	$f(2)+\int_2^{0} f'(x)dx=3-\left(-\frac{1}{2}\cdot 2\cdot 1\right)=4$
3	$f(2)+\int_2^{3} f'(x)dx=3-\left(1-\frac{\pi}{4}\right)=2+\frac{\pi}{4}\approx 2.8$
4	$f(2)+\int_2^{4} f'(x)dx=3-\left(2-\frac{\pi}{2}\right)=1+\frac{\pi}{2}\approx 2.6$

$\therefore$ the abs max value of f is 4.</td><td>fine solution</td></tr>
<tr><td>6</td><td>The only rel max of f is at $x=0$ because only there does f' change from positive to negative.

x	$f(x)$
-3	$f(2)+\int_2^{-3} f'(x)dx=3-\left(\frac{1}{2}\cdot 1\cdot 1\right)=2.5$
0	$f(2)+\int_2^{0} f'(x)dx=3-\left(-\frac{1}{2}\cdot 2\cdot 1\right)=4$
4	$f(2)+\int_2^{4} f'(x)dx=3-\left(2-\frac{\pi}{2}\right)=1+\frac{\pi}{2}\approx 2.6$

$\therefore$ the abs max value of f is 4.</td><td>fine solution</td></tr>
<tr><td>7</td><td>f has an abs max at $x=0$ because $f'>0$ for $x<0$ and $f'<0$ for $x>0$.
$f(0)=f(2)+\int_2^0 f'(x)\,dx=3-\left(-\frac{1}{2}\cdot 2\cdot 1\right)=4$.</td><td>Debatable! The open intervals are probably read as meaning within our domain. $f'>0$ is false at $x=3$. This might be forgiven.</td></tr>
</table>

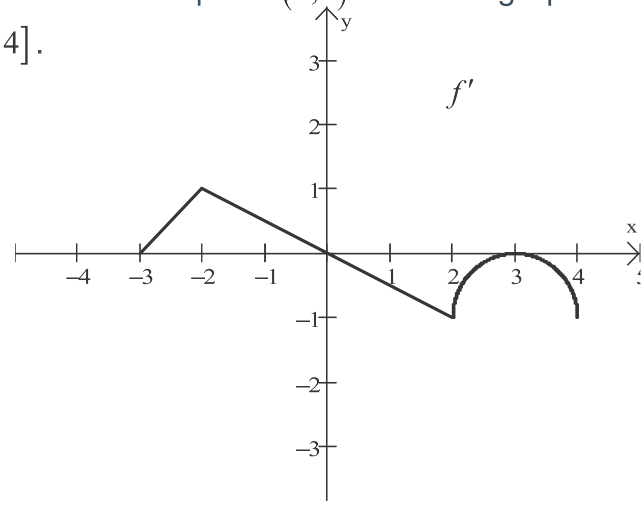
The graph of the continuous function f' consists of two line segments and a half circle as shown in the graph below. The point $(2,3)$ is on the graph of the function f whose domain is the closed interval $[-3,4]$.



(c) What is the absolute minimum value of f? Justify your answer.

	Explanation given	Awarded points on AP exam?
1	$f(-3) = f(2) + \int_2^{-3} f'(x)\,dx = 3 - \left(\frac{1}{2}\cdot 1\cdot 1\right) = 2.5$. $f(4) = f(2) + \int_2^{4} f'(x)\,dx = 3 - \left(2 - \frac{\pi}{2}\right) = 1 + \frac{\pi}{2} \approx 2.6$ f has no local min since f' does not change from negative to positive. Therefore the absolute minimum value of f is 2.5	*Both are fine solutions.*
2	x | $f(x)$ -3 | $f(2)+\int_2^{-3} f'(x)dx = 3-\left(\frac{1}{2}\cdot 1\cdot 1\right) = 2.5$ 0 | $f(2)+\int_2^{0} f'(x)dx = 3-\frac{1}{2}\cdot 2\cdot 1 = 4$ 3 | $f(2)+\int_2^{3} f'(x)dx = 3-\left(1-\frac{\pi}{4}\right) = 2+\frac{\pi}{4} \approx 2.8$ 4 | $f(2)+\int_2^{4} f'(x)dx = 3-\left(2-\frac{\pi}{2}\right) = 1+\frac{\pi}{2} \approx 2.6$ Therefore the absolute minimum value of f is 2.5.	

The graph of the continuous function f' consists of two line segments and a half circle as shown in the graph below. The point $(2,3)$ is on the graph of the function f whose domain is the closed interval $[-3,4]$.



(d) What are the *x*-coordinates of any points of inflection of f? Justify your answer.

	Explanation given	Awarded points on AP exam?
1	$x=-2, 2, 3$ because f' has extrema at these points and so f has POI there.	*Probably fine. Better to state which are maxima/minima.*
2	$x=-2, 2, 3$ because f' changes from increasing to decreasing at these points and so f has POI there.	*No. This is not true at $x=2$.*
3	$x=-2, 2, 3$ because f' changes increasing/decreasing behavior at these points and so f has POI there.	*Probably fine.*
4	$x=-2, 2, 3$ because f' changes from increasing to decreasing or vice versa at these points and so f has POI there.	*Probably fine.*
5	f' is increasing on $(-3,-2)$ and $(2,3)$, and f' is decreasing on $(-2,2)$ and $(3,4)$. Therefore f has POI at $x=-2, 2, 3$.	*Fine.*
6	f has POI where f' changes increasing/decreasing behavior. f has POI at $x=-2, 2, 3$.	*Debatable. This reads as a formula. Has it been tied to our problem?*
7	f has POI where f' changes increasing/decreasing behavior. Therefore f has POI at $x=-2, 2, 3$.	*probably okay. The "therefore" makes it a stronger justification.*
8	$x=-2, 2, 3$ because the slope of f' changes sign at these points and so f has POI there.	*Fine.*
9	$x=-2, 2, 3$ because f'' changes sign at these points and so f has POI there.	*Fine.*
10	$x=-2, 2, 3$ because the slope of f' changes from increasing to decreasing or from decreasing to increasing at these points and so f has POI there.	*False claim!*