

Name \_\_\_\_\_

More Derivatives

Use the following for 1-4:

$f(2) = -3$     $g(2) = 5$     $f'(2) = 4$     $g'(2) = -2$     $f'(5) = 6$

Find the derivative of the following at  $x = 2$ :

1. Find  $h'(2)$  if  $h(x) = \frac{f(x)}{g(x)}$

$h'(x) = \frac{g \cdot f' - f \cdot g'}{g^2}$     $h'(2) = \frac{g(2) \cdot f'(2) - f(2) \cdot g'(2)}{g(2)^2}$

$= \frac{5 \cdot 4 - (-3)(-2)}{5^2} = \frac{20 - 6}{25} = \frac{14}{25}$

2.  $f + g$     $f' + g' = f'(x) + g'(x)$   
 $= 4 + -2 = 2$

3.  $f \cdot g$

$- f \cdot g' + g \cdot f'$   
 $= f(2) \cdot g'(2) + g(2) \cdot f'(2)$   
 $= (-3)(-2) + (5)(4) = 6 + 20 = 26$

4.  $f(g(x))$

$= f'(g(x)) \cdot g'(x)$   
 $= f'(g(2)) \cdot g'(2)$   
 $= f'(5) \cdot g'(2) = 6 \cdot -2 = -12$

5. The position function of a moving particle is given by  $s(t) = 3t^3 + 2t - 1$ , where  $s$  is measured in feet and  $t$  is measured in seconds. Find the instantaneous acceleration at  $t = 2.5$  seconds.

$a(t) = s''(t)$     $s'(t) = 9t^2 + 2$   
 $s''(t) = 18t$   
 $a(2.5) = s''(2.5) = (18)(2.5) = \frac{18}{1} \left(\frac{5}{2}\right) = 45 \text{ ft/s}^2$

6. A coin is dropped from a height of 850 ft ( $s_0$ ).

$s(t) = -16t^2 + s_0$     $s(t) = -16t^2 + 850$

a) Find the average velocity on the interval  $[2, 3]$ .

$\frac{s(3) - s(2)}{3 - 2} =$

b) Find the instantaneous velocity at  $t = 1.5$  seconds.

$v(t) = s'(t) = -32t$   
 $v(1.5) = -32(1.5) = \frac{-32}{1} \left(\frac{3}{2}\right) = -48 \text{ ft/sec}$

For problems 7-89, let  $f(2) = 4$ ,  $f'(2) = 8$ ,  $g(2) = 5$ , and  $g'(2) = \frac{1}{2}$ .

7. Find  $h'(2)$  if  $h(x) = \frac{f(x)}{g(x)}$ .

$h'(x) = \frac{g \cdot f' - f \cdot g'}{g^2} = \frac{g(2) \cdot f'(2) - f(2) \cdot g'(2)}{(g(2))^2}$   
 $= \frac{5 \cdot 8 - 4 \cdot \frac{1}{2}}{5^2} = \frac{40 - 2}{25} = \frac{38}{25}$

8. Find  $h'(2)$  if  $h(x) = f(x)g(x)$ .

$h' = f \cdot g' + g \cdot f'$   
 $= f(2) \cdot g'(2) + g(2) \cdot f'(2)$   
 $= 4 \cdot \frac{1}{2} + 5 \cdot 8 = 2 + 40 = 42$

Find the derivative of each of the following functions.

9.  $f(x) = \sqrt[4]{x} + \sqrt[3]{x} = x^{1/4} + x^{1/3}$

$f'(x) = \frac{1}{4}x^{-3/4} + \frac{1}{3}x^{-2/3}$   
 $= \frac{1}{4\sqrt[4]{x^3}} + \frac{1}{3\sqrt[3]{x^2}}$

10.  $f(t) = \frac{2t}{t^3 + 1}$     $f'(t) = \frac{(t^3 + 1)(2) - (2t)(3t^2)}{(t^3 + 1)^2}$

$= \frac{2t^3 + 2 - 6t^3}{(t^3 + 1)^2} = \frac{2 - 4t^3}{(t^3 + 1)^2}$

11.  $f(x) = \sqrt[3]{x}(4x^2 + 1)$

$f'(x) = \frac{1}{3}x^{-2/3} \cdot 8x + \frac{1}{3}x^{-2/3}(4x^2 + 1) = 8x^{1/3} + \frac{4x^2 + 1}{3x^{2/3}}$

12.  $m(x) = 3 \csc x \cot x$

$m'(x) = -3 \csc^2 x - 3 \csc x \cot^2 x = -3 \csc x (\csc^2 x + \cot^2 x)$

13. If  $f(x) = -2x^4 + 4x^3 + 8x + 3$ , find  $f''(x)$ .

$f'(x) = -8x^3 + 12x^2 + 8$   
 $f''(x) = -24x^2 + 24x$

14. If  $f(x) = 3x^2 + 5x + \cos x$ , find  $f''(x)$ .

$f'(x) = 6x + 5 - \sin x$   
 $f''(x) = 6 - \cos x$

