Name Key

Test #7 Review

1. Find the limit, if it exists.

A. $\lim_{x\to\infty}\left(\frac{3x-2}{\sqrt{2x^2+1}}\right)$

BETC

B $\lim_{x\to-\infty}\left(\frac{3x-2}{\sqrt{2x^2+1}}\right)$

BETC

C. $\lim_{x\to\infty}\frac{x^2-4x+17}{x^3+9x^2+8}$

BOBO

answers

1. a. $\frac{3}{\sqrt{2}}$ b. $-\frac{3}{\sqrt{2}}$ c. 0

2. Find an equation of the tangent line to the graph of $y=(x-1)e^x+3\ln x+2$ at the point (1,2).

$y'=(x-1)e^x+e^x+\frac{3}{x}$

$y'(1)=0e^1+e^1+\frac{3}{1}=e+3$

$y-2=(e+3)(x-1)$

3. A bacteria cell is spherical in shape. If the radius of the cell is increasing at the rate of 0.01 micrometers per day. When it is 1.5 μm, what is the rate of increase of the volume of the cell at that time?

Find: $\frac{dV}{dt}$

When: $r=1.5$

Given: $\frac{dr}{dt}=.01$

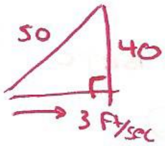
$V=\frac{4}{3}\pi r^3$

$\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$

$\frac{dV}{dt}=4\pi(1.5)^2(.01)$

$\frac{dV}{dt}=$

4. A child is flying a kite at a height of 40 ft, moving horizontally at a rate of 3 ft/sec. If the string is taut, at what rate is the string being released out when the length of the string released is 50 ft?



Find $\frac{dc}{dt}$

When $c=50\ (a=30)$

Given $b=40,\ \frac{da}{dt}=3$

$a^2+\overset{40}{\cancel{b^2}}=c^2$

$\cancel{2}a\frac{da}{dt}=\cancel{2}c\frac{dc}{dt}$

$30(3)=50\frac{dc}{dt}$

$\frac{dc}{dt}=\frac{90}{50}=\frac{9}{5}=1.8$ ft/sec

5. A spherical snowball melts so that the volume decreases at a rate of 1 cm^3/min, find the rate at which the diameter decreases when the diameter decreases when the diameter is 10 cm.

$r=\frac{1}{2}d$

Find $\frac{dd}{dt}$

When $d=10$

Given $\frac{dV}{dt}=-1$

$V=\frac{4}{3}\pi r^3$

$V=\frac{4}{3}\pi\left(\frac{d}{2}\right)^3=\frac{\pi d}{6}$

$\frac{dV}{dt}=\frac{\pi}{6}\frac{dd}{dt}$

$-1=\frac{\pi}{6}\frac{dd}{dt}$

$\frac{dd}{dt}=\frac{-6}{\pi}$ cm/min

6. Find all open intervals on which the function $f(x)=\frac{x}{x^2+x-2}$ is decreasing.

$f'(x)=\frac{(x^2+x-2)(1)-(x)(2x+1)}{(x^2+x-2)^2}$

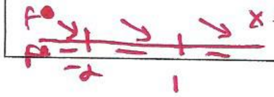
$=\frac{x^2+x-2-2x^2-x}{(x^2+x-2)^2}=\frac{-x^2-2}{(x^2+x-2)^2}=0$

$\cancel{x^2=-2}$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$x=-2 \quad x=1$



f is dec. on $(-\infty,-2)\cup(-2,1)\cup(1,\infty)$

7. Find an equation of the tangent line to the curve $y=x^3-3x-1$ at the point (2,1).

$y'=3x^2-3$

$y'(2)=3(4)-3=9=m$

$y-1=9(x-2)$

8. Determine increasing and decreasing intervals for the following functions.

a) $f(x)=2x^3+3x^2-12x$

$f'(x)=6x^2+6x-12=0$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$x=-2 \quad x=1$

f' + | − | +

f ↗ -2 ↘ 1 ↗

b) $f(x)=x^2-2x^3$

$f'(x)=2x-6x^2=0$

$2x(1-3x)=0$

$x=0 \quad x=1/3$

f' − | + | −

f ↘ 0 ↗ 1/3 ↘

answers

a) f inc. (-∞, -2) u (1,∞) f dec. (-2, 1)

b) f inc. $(0,\frac{1}{3})$ f dec. (-∞, 0) u $(\frac{1}{3},\infty)$

9. A.Find all critical numbers for the function

$f(x)=(9-x^2)^{\frac{3}{5}}$.

$f'(x)=\frac{3}{5}(9-x^2)^{-2/5}=0$

$\frac{3}{5\sqrt[5]{(9-x^2)^2}}=0$ $\quad 9-x^2=0$

$\boxed{x=\pm 3}$

B. Find the values of x that give the relative extrema for the function $f(x)=3x^5-5x^3$.

$f'(x)=15x^4-15x^2$

$15x^2(x^2-1)=0$

$x=0$

$x=\pm 1$

(a) Relative maximum: $x=0$; Relative minimum: $x=\sqrt{5/3}$

(b) Relative maximum: $x=-1$; Relative minimum: $x=1$

(c) Relative maxima: $x=\pm 1$; Relative minimum: $x=0$

(d) Relative maximum: $x=0$; Relative minima: $x=\pm 1$

(e) None of these

f' | -1 | 0 | 1

10. Be able to identify local maximums and minimums from the graph of the derivative.

max: deriv = 0; graph goes from above x-axis to below

min: deriv = 0; graph cross from below x-axis to above

11. How do you find vertical asymptotes?

Factor/cancel. Set denominator = 0. Solve

12. How do you find horizontal asymptotes?

I. BOBO, BOTU, BETC

or

II. Divide by highest-degreed variable on top/bottom

13. Which of the following functions has a horizontal asymptote at $y=-\frac{1}{2}$?

[✓ a.] $\frac{x^3}{1-2x^3}$ BETC

b. $\frac{x}{\sqrt{2x+1}}$ BOTU

c. $\frac{x^2-6x+1}{1+x^2}$ BETC $y=1$

d. $\frac{x-1}{2x^2+1}$ BOBO

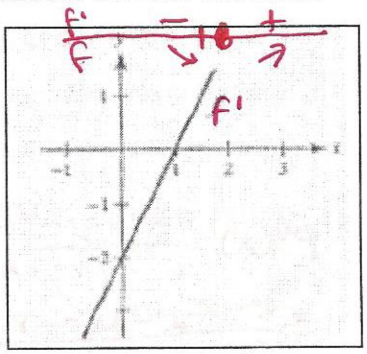
e. None of these

13.

14. How do you find critical numbers? How do you determine if they are local max or mins?

Find derivative. Set = 0 and set denominator = 0.

15. Use the graph of f' given in the figure to choose the true statement about f.



A. f has no local extrema.

B. f is increasing on the interval $(-\infty,\infty)$.

[✓ C.] f is decreasing on the interval $(-\infty,1)$.

D. f has a relative maximum at $x=1$.

E. None of these