First Derivative Test for Local Extrema

p. 278 - 286 (4.3) # 45

The **FIRST DERIVATIVE TEST** is used to find where a local (relative) maximum/minimum exists:

1) Determine the intervals where the function is inc/dec.
2. If the function changes from increasing to decreasing at a critical point (which exists), then that *x*-value is the relative maximum.
3. If the function changes from decreasing to increasing at a critical point (which exists), then that *x*-value is the relative minimum.

To find the relative maximum/minimum value:
Take the relative max/min *x*-value and substitute into the original function to get the max/min value.

**1. The function defined by $f(x) = x^3 - 3x^2$ for all real numbers has a relative maximum at $x =$?

2. State any local extrema for $y = \frac{1}{x^2}$.

3. Find the relative maximum value for $f(x) = (x^2 - 3)e^x$.

**3. What is the minimum value of $f(x) = x \ln x$?

Notes on Inc/Dec and Max/Min

1. For $g(x) = x^2 - 6x$, determine the
   a) intervals where g is increasing
   b) intervals where g is decreasing
   c) relative maximum value and where it occurs
   d) relative minimum value and where it occurs

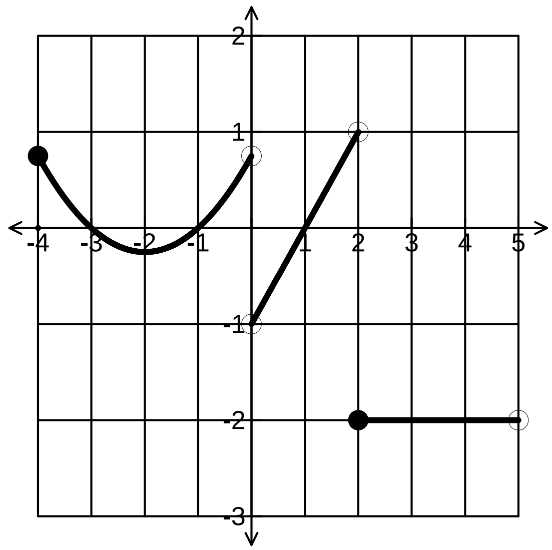
2. A function f is continuous on its domain [-3, 3] and f' has the following properties.

x	[-3,-1)	x=-1	(-1,2)	x=2	(2,3]
f'	-	0	-	undefined	+

   a) Find the intervals where f is decreasing.
   b) Find where any local extrema occur and justify your answer.
   c) Describe what is happening on f at $x = -1$.

3. For $h(x) = \sqrt[3]{x}(8 - x)$, determine
   a) the critical numbers
   b) the x-value(s) where any relative min occurs and justify
   c) the x-value(s) where any relative max occurs and justify

4. A function *f* is continuous on the interval [-4, 5]. The discontinuous function *f* ' is shown below. Using *f* ' answer the following questions.



a) State the interval(s) where *f* is decreasing and justify.

b) Find the *x*-coordinates of all local extrema and justify.