Rolle's Theorem

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(existence theorem)

Let f be continuous on the closed interval [a, b] and differentiable on the open interval (a, b). If $f(a) = f(b)$ then there exists at least one number c in (a, b) where $f'(x) = 0$.

If these conditions hold true, then there is at least 1 number between a & b so that the tangent line is horizontal.

Determine whether Rolle's Theorem can be applied. If so, find c. If not, tell why.

1. $f(x) = x^4 - 2x^2$ on [-2, 2].

**2. Let f be a function that is differentiable on the interval (1, 10). If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

I. f has at least 2 zeros.
II. The graph of f has at least one horizontal tangent.
III. For some c, $2 < c < 5$, $f(c) = 3$.

Class Notes

1. Determine whether Rolle's Theorem is valid $f(x) = 3 - |x - 2|$ for [-1, 5]. If so, find c. If not, tell why.

2. Find where the average rate of change equals the instantaneous rate of change for $y = \sqrt{x - 2}$ for [2, 6].

3. Determine the critical numbers for $y = \sqrt{x^2 - 2x - 15}$.

4. Locate the absolute extrema for $f(x) = \dfrac{3}{x - 1}$ for (1, 4].

5. Determine if Rolle's Theorem applies to $f(x) = (x - 3)(x + 1)^2$ for [-1, 3]. If so, find *c*. If not, tell why.

6. Suppose that *f*(0) = 4 and $f'(x) \leq 2$ for $x > 0$. Apply the MVT to the interval [0, 3] to prove that $f(3) \leq 10$.