Area Under a Curve

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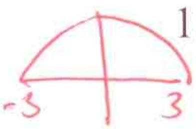
If $y = f(x)$ is nonnegative and integrable over a closed interval $[a, b]$, then the **area under the curve $y = f(x)$ from a to b** is given by

Area $= \int_a^b (f(x))\, dx$. This region is bounded by the x-axis, and the vertical lines $x = a$, and $x = b$ and is always *positive*.

However, a definite integral of an integrable $f(x)$ can represent other fields (profit/loss, water comsumption, entrance/exit into a store) and can be a positive or negative value.

$$\int_a^b (f(x))\, dx = \text{(area above the } x\text{-axis)} - \text{(area below the } x\text{-axis)}$$



1. Set up a definite integral that yields the area of the region denoted by $f(x) = \sqrt{9 - x^2}$. Solve your integral.

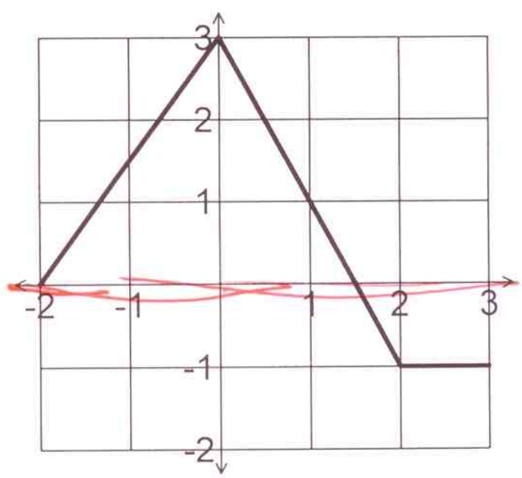
$y = \sqrt{9-x^2}$

$y^2 = 9 - x^2$

$x^2 + y^2 = 9$

$\int_{-3}^{3} \sqrt{9-x^2} = \frac{1}{2}(\pi)(3)^2 = \frac{9}{2}\pi$

≈ 14.137



2. In the graph to the right, the function f is defined for $-2 \le x \le 3$. What is the value of $\int_{-2}^{3} (f(x))\, dx$?

$= \int_{-2}^{1.5} f(x) - \int_{1.5}^{3} f(x)$

$= \frac{1}{2}(3.5)(3) - \frac{1}{2}(1.5+1)$

$= \frac{1}{2}\left(\frac{21}{2}\right) - \frac{1}{2}\left(\frac{5}{2}\right) = \frac{21}{4} - \frac{5}{4} = \frac{16}{4} = 4$