

Riemann Sums - Unequal Widths

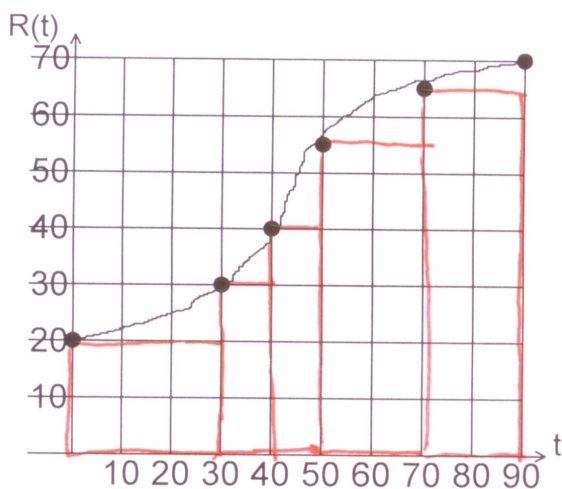
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Let f be continuous and non-negative on the interval $[a, b]$, then a Riemann Sum can be used to find an approximate area of f under the curve if the widths of the rectangles are unequal by:

1. Area = $\underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \dots$ where n is # of blanks = # of rectangles
2. Each blank has (ht.) (Δx) of each rectangle where (Δx) is different for each rectangle.
3. The heights are determined by the problem situation (upper, midpt., etc.).

**calc.(FR) The rate of fuel consumption (in gallons per minute) recorded during a plane flight is given by a twice differentiable function R of time t , min.



| t | $R(t)$ |
|-----|--------|
| 0 | 20 |
| 30 | 30 |
| 40 | 40 |
| 50 | 55 |
| 70 | 65 |
| 90 | 70 |

1. Approximate the value of the total fuel consumption, using a left Riemann sum with the 5 subintervals indicated in the table above.

$$S_L = (30)(20) + (10)(30) + (10)(40) + (20)(55) + (20)(65) = \cancel{2500}^{3700} \text{ gal.}$$

2. Is this numerical approximation less than the value of the exact area?
Explain your reasoning.

Since $R'(t) > 0$, the left Riemann sum is less than the exact value.