Trapezoid Rule

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Let f be continuous on [a,b] with n subintervals of equal width.

Then, the trapezoidal rule for approximating $\int_a^b (f(x))\,dx$ is

$$\frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{(n-1)}) + f(x_n)\right]$$

and as $n \to \infty$, the right side approaches $\int_a^b (f(x))\,dx$.

**Used because can't use Fundamental Thm. of Calculus, so must approximate...antiderivatives aren't always elementary functions.

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1. Use the trapezoidal rule to approximate $\int_0^2 x^3\,dx$ for $n = 4$.

$\frac{b-a}{n} = \frac{2}{4} = \frac{1}{2}$

$S_T = \frac{2-0}{2(4)}\left[0 + 2\left(\frac{1}{2}\right)^3 + 2(1)^3 + 2\left(\frac{3}{2}\right)^3 + (2)^3\right] = \frac{1}{4}\left[\frac{1}{4} + 2 + \frac{27}{4} + 8\right] = \frac{1}{4}(17)$

$= \boxed{\frac{17}{4}}$

**calc. 2. The function f is continuous on [2, 8] and has values that are given in the table below. Using the subintervals [2, 5], [5, 7], and [7, 8], what is the trapezoidal approximation of $\int_2^8 (f(x))\,dx$?

3 2 1

x	2	5	7	8
f(x)	10	30	40	20

$S_T = \frac{1}{2}(3)(10 + 30) + \frac{1}{2}(2)(30 + 40) + \frac{1}{2}(1)(40 + 20)$

$= 60 + 70 + 30 = \boxed{160}$