

## Antiderivatives

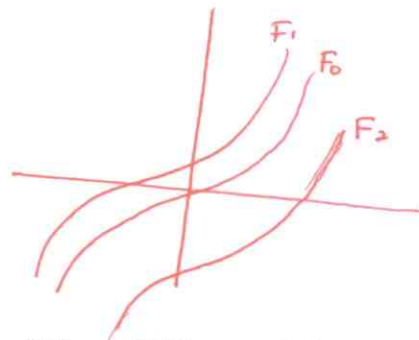
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1. A function  $F(x)$  is an **antiderivative** of a function  $f(x)$  if  $F'(x) = f(x)$  for all  $x$  in the domain of  $f$ . The process of finding an antiderivative is **antidifferentiation**.
2. The family of all antiderivatives of a function  $f(x)$  is the **indefinite integral** of  $f$  with respect to  $x$  and it denoted by  $\int(f(x)) dx$ .
3. If  $F$  is any function such that  $F'(x) = f(x)$ , then  $\int(f(x)) dx = F(x) + C$  is called the general solution and  $C$  is called the **constant of integration** (an arbitrary constant).

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1. State three functions  $F_1, F_2, F_3$  whose derivatives are  $f(x) = 3x^2$ . Now sketch all three of your functions on the same coordinate plane. Compare these with  $f(x) = 3x^2$ .

$$\begin{aligned}F_0 &= x^3 \\F_1 &= x^3 + 1 \\F_2 &= x^3 - 5 \\F_3 &= x^3 + 100\end{aligned}$$



Same curve.  
General solution:  
 ~~$F(x) = x^3 + C$~~

2. Find a general solution of the differential equation  $\frac{dy}{dx} = 3x^{-4}$  and check by differentiation.

$$\int dy = \int 3x^{-4} dx$$

$$y = \frac{3x^{-3}}{-3} + C$$

$$y = -x^{-3} + C$$

$$y = -\frac{1}{x^3} + C$$

$$\begin{aligned}\frac{d}{dx} \left[ -\frac{1}{x^3} + C \right] &= \frac{d}{dx} \left[ -x^{-3} + C \right] \\ &= -(-3)x^{-4} = 3x^{-4} \checkmark\end{aligned}$$