Fundamental Theorem of Calculus

p. 377 - 383 (5.4) # 63

If f is a continuous function on [a, b] and F is an antiderivative of f on [a,b], then

$$\int_a^b (f(x))\,dx = F(b) - F(a)$$

Graphically this means: the signed area bounded by $x = a,\ x = b,$ $y = f(x)$, and the x-axis.

Evaluate.

**1. $\int_1^2 (4x^3 - 6x)\,dx = x^4 - 3x^2 \Big|_1^2$ $= 6$

**2. $\int_0^{\pi/4} \sin(x)\,dx = 1 - \frac{\sqrt{2}}{2}$ or $\frac{2-\sqrt{2}}{2}$

**3. $\int_0^1 e^{(-4x)}\,dx$ $= \boxed{\frac{-1}{4}\left(\frac{1}{e^4} - 1\right)}$ $\approx .2454$

4. $\int_{(\ln 2)}^3 5e^x\,dx$ $= 5(e^3 - 2)$ $= \boxed{5e^3 - 10}$

**5. $\int_1^2 \frac{x-4}{x^2}\,dx$ $= \boxed{\ln 2 - 2}$

**6. If $\int_a^b (f(x))\,dx = a + 2b$, then $\int_a^b ((f(x) + 5))\,dx = ?$

$= \int_a^b f(x)dx + 5\int_a^b dx = a + 2b + 5x\Big|_a^b = \boxed{7b - 4a}$

**7. Using the substitution $u = 2x + 1$, $\int_0^2 \left(\sqrt{2x+1}\right)\,dx$ is equal to

(A) $\frac{1}{2}\int_{-1/2}^{1/2} \sqrt{u}\,du$ (B) $\frac{1}{2}\int_0^2 \sqrt{u}\,du$ (C) $\frac{1}{2}\int_1^5 \sqrt{u}\,du$ (D) $\int_0^2 \sqrt{u}\,du$ (E) $\int_1^5 \sqrt{u}\,du$

$u = 2x + 1$
$2(0) + 1 = 1$
$2(2) + 1 = 5$

$u = 2x + 1$
$du = 2dx$
$dx = \frac{1}{2}du$

Notes on FTC

1. $\int_{(-1)}^{3} (4x - 2)\, dx$ $= 8$

2. $\int_{-\pi/3}^{\pi/3} (\sec\theta \tan\theta)\, d\theta$ $= 0$

3. $\int_{1/2}^{1} \frac{1}{2x}\, dx$ $= -\frac{1}{2} \ln \frac{1}{2}$

4. $\int_{0}^{1} \left(x^2 + \sqrt{x}\right) dx$ $= 1$

5. $\int_{\pi/6}^{5(\pi)/6} \frac{\csc^2\theta}{2}\, d\theta$ $= \sqrt{3}$

6. $\int_{0}^{3} \left(e^t - t\right) dt$ $= e^3 - \frac{11}{2}$