Second Fundamental Theorem of Calculus

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If f is continuous on an interval [a, b], then the function $F(x) = \int_a^x f(t)\,dt$ has a derivative at every point x in [a, b], and

$$\frac{dF}{dt} = \frac{d}{dx}\left[\int_a^x (f(t))\,dt\right] = f(x)$$

Similarly,

$$\frac{d}{dx}\left[\int_a^u (f(t))\,dt\right] = \frac{du}{dx}\cdot f(u)$$

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1. For $F(x) = \int_2^x \left(\frac{1}{3t^2}\right) dt$, find

a) $F(x) = \frac{1}{3}\int_2^x t^{-2}\,dt$ b) $F(2) = \int_2^2 \left(\frac{1}{3t^2}\right) dt$ c) $F'(x) = \frac{1}{3x^2}$ d) $F'(2) = \frac{1}{3(2)^2} = \frac{1}{12}$

$= \frac{1}{3}\left(-\frac{1}{t}\right)\Big|_2^x$

$= -\frac{1}{3}\left(\frac{1}{x} - \frac{1}{2}\right)$

$= 0$

**2. Evaluate $\frac{d}{dx}\int_2^x \left(\sqrt{1+t^2}\right) dt$ $= \sqrt{1+x^2}$

3. Find $F'(x)$ if $F(x) = \int_{-\pi/3}^{x^3} (\cos(t))\,dt$ $\cos(x^3)(3x^2)$

Notes on 2nd FTC

1. Find $\int_0^x (\sin(t))\, dt$. $= -\cos t \Big|_0^x = -\cos x - (-\cos 0)$

$= 1 - \cos x$

2. If $f(x) = \int_1^x \left(\sqrt{1+t^3}\right) dt$, what is $f'(x)$?

$\sqrt{1+x^3}$

3. If $f(x) = \int_1^x \left(\sqrt{1+t^3}\right) dt$, what is $f'(x)$?