

Integration of $\ln(x)$

p. 386 - 392 (5.52)

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$$\int \frac{1}{x} dx = \ln(|x|) + C$$

$$\int \frac{1}{u} du = \ln(|u|) + C \text{ (BE CAREFUL, must do u-subst.)}$$

Integrate:

$$\begin{aligned} 1. \int \frac{3e^x}{1+e^x} dx & \quad \begin{aligned} u &= 1+e^x \\ du &= e^x dx \\ dx &= \frac{du}{e^x} \end{aligned} & \quad \begin{aligned} &= 3 \int \frac{e^x}{u} \frac{du}{e^x} = 3 \int \frac{1}{u} du \\ &= 3 \ln|u| + C \\ &= 3 \ln|1+e^x| + C \end{aligned} \end{aligned}$$

$$\begin{aligned} **2. \int \frac{x-4}{x^2} dx & \quad \begin{aligned} u &= x^2 \\ du &= 2x dx \\ dx &= \frac{du}{2x} \end{aligned} & \quad \int \frac{x-4}{x^2} dx \\ &= \int \frac{x}{x^2} dx - \int \frac{4}{x^2} dx = \int \frac{1}{x} dx - 4 \int x^{-2} dx = \ln|x| - \frac{4x^{-1}}{-1} + C \\ &= \ln|x| + \frac{4}{x} + C \end{aligned}$$

$$\begin{aligned} 3. \int \frac{x}{2(x+1)^2} dx &= \frac{1}{2} \int \frac{u-1}{u^2} du \\ u &= x+1 \quad x = u-1 \\ du &= dx &= \frac{1}{2} \int \left(\frac{1}{u} - \frac{1}{u^2} \right) du \\ &= \frac{1}{2} \left[\ln|u| + \frac{1}{u} \right] + C = \frac{1}{2} \ln|x+1| + \frac{1}{x+1} + C \end{aligned}$$