Differentiability Implies Continuity

p. 155 - 165 (2.8)

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1. If a function is differentiable at $x = c$, then it is continuous at $x = c$. Continuity means $f(c) = \lim_{x \to c^+} (f(x)) = \lim_{x \to c^-} (f(x))$
2. It is possible for a function to be continuous at $x = c$ and **not** differentiable at $x = c$.

Differentiable means you can find the slope at that point or that the derivative does exist.

**1. Let f be a function such that $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = 5$.

defn of deriv ⇒ deriv @ x=2 is 5

Which of the following must be true?

I. f is continuous at $x = 2$. ✓
II. f is differentiable at $x = 2$. ✓
III. The derivative of f is continuous at $x = 2$.

(A) I only (B) II only (C) I and II only
(D) I and III only (E) II and III only

**(FR) 2. Let f be a function be defined by

$$f(x) := \begin{cases} 2x - x^2 & x \le 1 \\ x^2 + kx + p & x > 1 \end{cases}$$

For what values of k and p will f be continuous and differentiable at $x = 1$?

continuous:
$2(1) - 1 = 1 + k(1) + p$
$2 - 1 = 1 + k + p$
$k + p = 0$

differentiable:
$2 - 2x = 2x + k$ @ $x = 1$
$2 - 2 = 2 + k$
$k = -2$

$k + p = 0$
$-2 + p = 0$
$p = 2$