Normal Lines

p. 183-193 (3.1) # 17

A normal line to a curve at a point is the line perpendicular to the tangent at that point.

The slope of a normal line is the opposite, reciprocal slope of the tangent line.

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1. Find the equation of the line normal to $h(x) = \dfrac{3}{x^2}$ at $x = -4$.

$h(-4) = \frac{3}{16}$ $h(x) = 3x^{-2}$

$h'(x) = \frac{-6}{x^3}$ $h'(-4) = \frac{-6}{(-4)^3} = \frac{-6}{-64} = \frac{3}{32}$ → normal line slope $= \frac{-32}{3}$

$\boxed{y - \frac{3}{16} = -\frac{32}{3}(x+4)}$

2. Find the slope of the line normal to the graph of $f(x) = 4\sqrt{x} - 2x$ at (9, -6).

$f'(x) = 4(\frac{1}{2})x^{-1/2} - 2 = \frac{2}{\sqrt{x}} - 2$

$f'(9) = \frac{2}{\sqrt{9}} - 2 = \frac{2}{3} - 2 = \frac{2}{3} - \frac{6}{3} = -\frac{4}{3}$

$\boxed{\text{normal line slope} = 3/4}$

3. If it exists, determine the equation of the line normal to the following function at the breakpoint.

continuous: ?

$\lim_{x \to 1^-} g(x) = 4$

$\lim_{x \to 1^+} g(x) = 4$ yes

(1, 4)

$$g(x) := \begin{cases} x^2 + 3 & x < 1 \\ 2x + 2 & x \geq 1 \end{cases}$$

differentiable @ $x = 1$?

$\rightarrow 2x$ @ $x = 1 \Rightarrow 2 = 2$

$\rightarrow 2$

$m = 2$ ← yes

normal line $m = -\frac{1}{2}$

$\boxed{y - 4 = -\frac{1}{2}(x - 1)}$

4. Suppose p and q are differentiable functions with the values in the table below. Find the equation of the line normal to $r(x)$ at $x = -2$ provided $r(x) = p(x) - 3q(x)$.

$r(-2) = p(-2) - 3q(-2)$
$= 5 - 3(7) = 5 - 21 = -16$
$(-2, -16)$

x	$p(x)$	$q(x)$	$p'(x)$	$q'(x)$
-2	5	7	-1	6

$r'(x) = p'(x) - 3q'(x)$

$r'(-2) = p'(-2) - 3q'(-2)$

$= (-1) - 3(6)$

$= -1 - 18 = -19$

normal line slope $= \frac{1}{19}$

$\boxed{y + 16 = \frac{1}{19}(x+2)}$