

Implicit Differentiation

p. 232 - 238 (3.6)

30

1. Differentiate both sides with respect to x .
2. Collect $\frac{dy}{dx}$ terms on one side = all other terms on the other side
3. Factor out $\frac{dy}{dx}$.
4. Solve for $\frac{dy}{dx}$.

1. Find $\frac{dy}{dx}$ for $y^4 + x^3y^5 - 2x^7 = 13$.

$$4y^3 \frac{dy}{dx} + x^3 5y^4 \frac{dy}{dx} + 3x^2 y^5 - 14x^6 = 0$$

$$4y^3 \frac{dy}{dx} + 5x^3 y^4 \frac{dy}{dx} = 14x^6 - 3x^2 y^5$$

$$\frac{dy}{dx} (4y^3 + 5x^3 y^4) = 14x^6 - 3x^2 y^5$$

$$\frac{dy}{dx} = \frac{14x^6 - 3x^2 y^5}{4y^3 + 5x^3 y^4}$$

2. Find the line of tangency and normal line to the curve $x^2 - xy + y^2 = 7$ at the point $(-1, 2)$.

$$2x - (x \frac{dy}{dx} + y) + 2y \frac{dy}{dx} = 0$$

$$m = \frac{2 - 2(-1)}{2(-1) - (-1)} = \frac{2+2}{-4+1} = \frac{4}{-3} = -\frac{4}{3}$$

$$-x \frac{dy}{dx} + 2y \frac{dy}{dx} = y - 2x$$

$$y - 2 = \frac{4}{-3}(x + 1)$$

$$\frac{dy}{dx} = \frac{y - 2x}{2y - x}$$

$$y - 2 = -\frac{5}{4}(x + 1)$$

3. Find the instantaneous rate of change at $(1, 1)$ for

$$x + 3xy - 2y^2 = 2.$$

$$1 + 3x \frac{dy}{dx} + 3y - 4y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-1 - 3y}{3x - 4y}$$

$$\frac{dy}{dx} (3x - 4y) = -1 - 3y$$

$$m = \frac{-1 - 3(1)}{3(1) - 4(1)} = \frac{-1 - 3}{3 - 4} = \frac{-4}{-1} = 4$$

**4. If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{(d(x))^2}$ at the point

$(4, 3)$?

$$2x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y} \quad m = -\frac{4}{3}$$

$$\frac{d^2y}{dx^2} = \frac{y(-1) - (-x) \frac{dy}{dx}}{y^2}$$

$$\Rightarrow \frac{-(-3) + (4)(-\frac{4}{3})}{3^2}$$

$$= \frac{-3 - \frac{16}{3}}{9} = \frac{-\frac{25}{3}}{9} = \frac{-25}{27}$$