# Derivative of an Inverse Function

p. 232 - 238 (3.6) # 31

Let $f$ be a differential function. If $f$ possesses an inverse function $f^{-1}$, then $f^{-1}$ is differentiable at any $x$ for which $f'\left(f^{-1}(x)\right) \neq 0$. Therefore, $\left(f^{(-1)}\right)'(x) = \dfrac{1}{f'\left(f^{(-1)}(x)\right)}$.

To find $f'\left(f^{-1}(b)\right)$ for a point $(a, b)$ on $f(x)$.

1. Find $f'$.
2. If you are only given $b$, set $b = f(x)$ to find $a$.
3. Find $f'(a)$.
4. $f'\left(f^{-1}(b)\right) = \dfrac{1}{f'(a)}$

---

1. If $g$ is the inverse function of $f$ and $f(3) = 2$, find the value of $g'(2)$ for $f(x) = \dfrac{x^3}{4} + x - 1$.

$f(3,2)$
$g(2,3)$

$g'(2) = \dfrac{1}{f'(3)} = \dfrac{1}{31/4} = \boxed{4/31}$

$f'(x) = \dfrac{3x^2}{4} + 1$

$f'(3) = \dfrac{3(9)}{4} + 1 = \dfrac{27}{4} + 1 = \dfrac{31}{4}$

2. Let $f$ be the function defined by $f(x) = x^5 + 2x - 1$. If $g(x) = f^{-1}(x)$ and $(2, 1)$ is on $f$, what is the value of $g'(1)$?

$g'(1) = \dfrac{1}{f'(2)}$

$f'(x) = 5x^4 + 2$

$f'(2) = 5(2)^4 + 2 = 82$

$g'(1) = \dfrac{1}{f'(2)} = \dfrac{1}{82}$

**3. Let $f$ be the differentiable function such that $f(3)=15$, $f(6)=3$, and $f'(3)= -8$, and $f'(6)= -2$. The function $g$ is differentiable and $g(x) = f^{-1}(x)$ for all $x$. What is the value of $g'(3)$?

$(3, 15)$
$(6, 3)$

$g'(3) = \dfrac{1}{f'(6)} = \dfrac{1}{-2} = \boxed{-\dfrac{1}{2}}$