Natural Logarithmic Function

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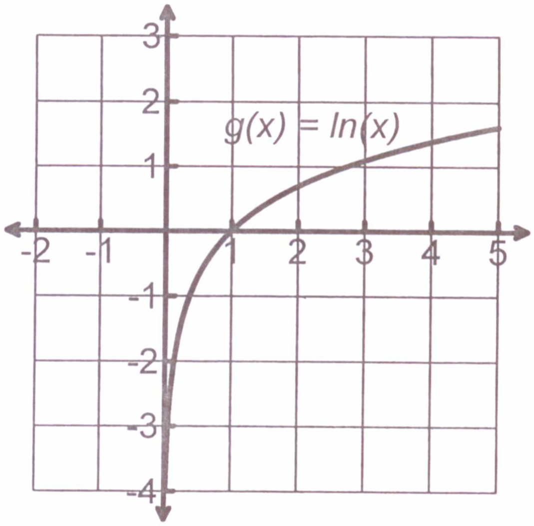

1. The inverse of $g(x) = \ln(x)$ is $g^{-1}(x) = e^x$
2. Continuous, and increasing everywhere *(monotonic)*
3. $\lim\limits_{x \to (-\infty)} (g(x)) = \text{DNE}$ and $\lim\limits_{x \to \infty} (g(x)) = \infty$
4. Must know..... $\ln 1 = 0$ and $e^{(\ln x)} = x$

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x} \qquad \frac{d}{dx}(\ln(u)) = \frac{du}{u}, \text{ for } u > 0$$

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1. Find $v'(t)$ for $v(t) = \ln(t^2 + 3)$.

$v'(t) = \dfrac{2t}{t^2 + 3}$

2. If $y = 4x \ln(5x^4)$, find y'.

$4 \quad \dfrac{20x^3}{5x^4} = \dfrac{4}{x}$

$y' = 4x\left(\frac{4}{x}\right) + 4 \ln(5x^4)$

$= 16 + 4 \ln(5x^4)$

3. Differentiate $h(x) = \dfrac{\ln(\tan(x))}{3\pi}$.

$h'(x) = \dfrac{1}{3\pi} \dfrac{\sec^2 x}{\tan x}$

**4. Find the slope of the line normal to the graph of $y = 2\ln(\sec x)$ at $x = \dfrac{\pi}{4}$.

$y' = 2\left(\dfrac{\sec x \tan x}{\sec x}\right) = 2\tan x$ $\quad m = -\frac{1}{2}$

$y'\left(\frac{\pi}{4}\right) = 2\tan\left(\frac{\pi}{4}\right) = 2$

**(FR) 5. Write an equation for the line tangent to the graph of $f(x) = -2 + \ln(x^2)$ at $x = 1$.

$f(1) = -2 + \ln(1)^2 = -2$

$f'(x) = \dfrac{2x}{x^2} = \dfrac{2}{x}$ $\quad f'(1) = 2$ $\quad \boxed{y + 2 = 2(x - 1)}$

**6. If $f(x) = \ln(x + 4 + e^{-3x})$, then $f'(0)$ is _____.

$f'(x) = \dfrac{1 - 3e^{-3x}}{x + 4 + e^{-3x}}$ $\quad f'(0) = \dfrac{1 - 3e^0}{0 + 4 + e^0} = \dfrac{1-3}{4+1} = \boxed{\dfrac{-2}{5}}$