Logarithmic Differentiation

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1. Take the natural log of both sides of an equation
2. Use properties of logs to simplify
3. Differentiate both sides (with respect to x)
4. Get all of the $\frac{dy}{dx}$ terms on one side of the equal sign
5. Get all of the other terms on the other side
6. Factor out $\frac{dy}{dx}$
7. Isolate $\frac{dy}{dx}$ by itself (by dividing by the other term)
8. Replace y with what it was originally equivalent to.

Find $\frac{dy}{dx}$ by using logarithmic differentiation

1. $y = \sqrt{(x-1)(x-2)(x-3)}$

$\ln y = \ln\left[(x-1)(x-2)(x-3)\right]^{1/2}$

$\ln y = \frac{1}{2}\left[\ln(x-1) + \ln(x-2) + \ln(x-3)\right]$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x-1} + \frac{1}{x-2} + \frac{1}{x-3}\right] \cdot y$

$\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x-1} + \frac{1}{x-2} + \frac{1}{x-3}\right]\left[\sqrt{(x-1)(x-2)(x-3)}\right]$

2. $y = \sqrt{x}(\cdot e)^{x^2}(x^2+1)^{10}$

$\ln y = \ln\left(x^{1/2} \cdot e^{x^2} \cdot (x^2+1)^{10}\right)$

$\ln y = \frac{1}{2}\ln x + x^2 \ln e + 10\ln(x^2+1)$

$\ln y = \frac{1}{2}\ln x + x^2 + 10\ln(x^2+1)$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x}\right) + 2x + 10\left(\frac{2x}{x^2+1}\right)$

$\frac{dy}{dx} = \left[\frac{1}{2x} + 2x + \frac{20x}{x^2+1}\right]\left[\sqrt{x} \cdot e^{x^2}(x^2+1)^{10}\right]$