Linear Approximations

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Sometimes, we might not be able to calculate the equation of a tangent line at a specific point, but we can find the equation of the tangent line at a nearby point. If this is the case, we can use a *linear approximation* to get a value that's really close to the actual answer that we are not able to find.

To do this:

1. Find the equation of the tangent line at a relatively close point.
2. Substitute in the nearby x-value to find the approximation for y.
3. If the function is **concave up**, your approximation is an **under-estimate**.
4. If the function is **concave down**, your approximation is an **over-estimate**.

**1. The function f is twice differentiable with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at x = 2?

$(2,1)$ $m=4$

$y-1=4(x-2)$

$f(\cancel{x}) = 4x-7$

$f(1.9) = 4(1.9)-7$

$= 7.6-7 = \boxed{.6}$

**2(FR).

x	-1.5	-1	-0.5	0	0.5	1	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval [-1.5, 1.5]. The second derivative of f has the property that $f'' > 0$ for [-1.5, 1.5], which means f is concave up on that interval.

(curving upward)

$f(1) = -4$ $f'(1) = 5$

$y+4 = 5(x-1)$

$y = 5x-9$

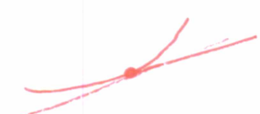
$f(x) = 5x-9$

Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

$f(1.2) = 5(1.2) - 9$

$= 6 - 9$

$= -3$



Since f is increasing @ x=1 and f is concave up, then $f(1.2) \approx -3$ is an underestimate for the actual value of f@ x=1.2.