Related Rates/Change Variables

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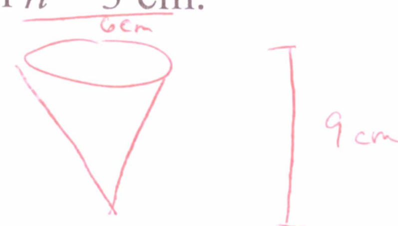
Use this method usually when some rate or ratio is given for one variable in the problem.

1. Draw, then determine "find", "when", and "given".
2. Using the ratio, solve 1 variable in terms of another.
3. Substitute that info into the relating equation first, THEN differentiate.
4. Plug in the "given" and "when", then solve for the "find".

A inverted cone has a height of 9 cm and a diameter of 6 cm. It is leaking water at the rate of 1 cm^3/min. Find the rate at which the water level is dropping when $h = 3$ cm.

$$V = \frac{1}{3}\pi r^2 h$$



Find : $\frac{dh}{dt}$

When: $h = 3\text{cm}$ ($\Rightarrow r = 1$)

$\frac{r}{h} = \frac{3}{9}$

$r = \frac{1}{3} h$

Given $\frac{dV}{dt} = -1 \text{ cm}^3/\text{min}$

$r = 3\text{cm}$

$h = 9\text{cm}$

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}\left(\frac{h}{3}\right)^2 h$

$V = \frac{\pi}{27} h^3$

$\frac{dV}{dt} = \frac{\pi}{27} 3h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{\pi}{9} h^2 \frac{dh}{dt}$

$-1 \text{ cm}^3/\text{min} = \frac{\pi}{9}(3)^2 \frac{dh}{dt}$

$-1 = \pi \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = -1/\pi \text{ cm/min}}$