

Finding Extrema Using Critical Numbers

p. 278 - 286 (4.3)

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Critical Numbers: Let f be defined at c . If $f'(c) = 0$ or if f' is undefined at c , then c is a critical number. To find the critical point, find the y -value of the function at c or $f(c)$.

To find extrema of f for $[a,b]$:

1. Find the critical numbers of f in (a,b)
2. Evaluate f at each critical number in (a,b)
3. Evaluate f at each endpoint of $[a,b]$
4. The least value = minimum and the greatest value = maximum

** (calc.) 1. For $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$ how many critical values does f have on the interval $(0, 10)$? 3

2. Find the extrema values on $f(x) = \frac{9(x^2 - 3)}{x^3}$ for $[-4, -1]$.

$$f'(x) = \frac{(x^3)(18x) - (9x^2 - 27)(3x^2)}{(x^3)^2} = \frac{3x^2(6x^2 - 9x^2 + 27)}{x^6}$$

$$\frac{3x^2(-3x^2 + 27)}{x^6} = \frac{9(-x^2 + 9)}{x^4} = 0 \quad x = \pm 3$$

easier to use $f = \frac{9x^2}{x^3} - \frac{27}{x^3}$

$f(-4) = -\frac{117}{64}$

 $f(-1) = 18$ max Value
 $f(-3) = -2$ min Value

**3. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the interval $[-2, 4]$ is 28.

$$f'(x) = 3x^2 - 6x = 0$$

$$3x(x - 2) = 0$$

$$x = 0 \quad x = 2$$

$f(-2) = -8$ ← global min
 $f(0) = 12$
 $f(2) = 8$
 $f(4) = 28$ ← global max

$64 - 48 + 12$

