First Derivative Test for Local Extrema

p. 278 - 286 (4.3) # 45

The **FIRST DERIVATIVE TEST** is used to find where a local (relative) maximum/minimum exists:

1) Determine the intervals where the function is inc/dec.
2. If the function changes from increasing to decreasing at a critical point (which exists), then that x-value is the relative maximum.
3. If the function changes from decreasing to increasing at a critical point (which exists), then that x-value is the relative minimum.

To find the relative maximum/minimum value:
Take the relative max/min x-value and substitute into the original function to get the max/min value.

**1. The function defined by $f(x) = x^3 - 3x^2$ for all real numbers has a relative maximum at $x =$?

$f'(x) = 3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0 \quad x = 2$

$+ \quad - \quad +$; 0, 2

$x = 0$

2. State any local extrema for $y = \dfrac{1}{x^2}$. $= x^{-2}$

$y' = -2x^{-3} = \dfrac{-2}{x^3}$

$+ \quad -$; 0

max @ $x=0$ but y DNE @ $x=0$, so no extrema.

3. Find the relative maximum value for $f(x) = (x^2 - 3)e^x$.

$f'(x) = (x^2-3)(e^x) + (e^x)(2x) = 0$

$e^x(x^2 - 3 + 2x) = 0$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3 \quad x = 1$

$+$ (max) $\quad - \quad +$; -3, 1

$f(-3) = (9-3)e^{-3}$

$= \boxed{\dfrac{6}{e^3}}$

**3. What is the minimum value of $f(x) = x \ln x$?

$f'(x) = x\left(\dfrac{1}{x}\right) + \ln x = 0$

$1 + \ln x = 0$

$\ln x = -1$

$e^{-1} = x$

$- \quad +$; e^{-2}, e^{-1}, e^{0}

$f\left(\dfrac{1}{e}\right) = \dfrac{1}{e}\ln e^{-1}$

$= \boxed{-\dfrac{1}{e}}$