# Determine Concavity and POI's

p. 278 - 286 (4.3)

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To find the intervals of concavity for a function $y$:

1. Find $y''$ then find where $y'' = 0$ or where $y''$ is undefined. (PPOI's)
2. Put these PPOI's on a number line and test a number within the intervals into $y''$.
3. If $y'' > 0$, then $y$ is concave up. If $y'' < 0$, then $y$ is concave down.

A point where the graph of a function has a tangent line and where the concavity changes is a **point of inflection** (POI).

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**1. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for what intervals?

$y' = 12x^3 - 48x^2 + 48x$

$y'' = 36x^2 - 96x + 48 = 0$

$3x^2 - 8x + 4 = 0$

$(x - 2/3)(x - 2) = 0$

$x = 2/3 \quad x = 2$

$f''$: $+$ | $-$ | $+$

$f$: $\cup$ $2/3$ $\cap$ $2$ $\cup$

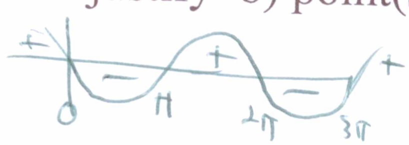
c.c. down $(2/3, 2)$ since $y'' < 0$.

2. For $f(x) = \sin(x) + 2$ on $[0, 3\pi]$ determine the a) intervals where $f(x)$ is concave up and justify b) point(s) of inflection and justify.

$f'(x) = \cos x$

$f''(x) = -\sin x$

$x = 0, \pi, 2\pi, 3\pi$




CC up on $(\pi, 2\pi)$ since $f'' > 0$ on $(\pi, 2\pi)$

POI $(0, 2), (\pi, 2), (2\pi, 2), (3\pi, 2)$

3. Determine the intervals where $g(x) = \sqrt[3]{2x - 1}$ is concave down and concave up and state the point(s) of inflection.

$g'(x) = \frac{1}{3}(2x-1)^{-2/3}(2) = \frac{2}{3}(2x-1)^{-2/3}$

$g''(x) = -\frac{4}{9}(2x-1)^{-5/3}(2) = \frac{-8}{9}(2x-1)^{-5/3} = 0$

$2x - 1 = 0$

$x = 1/2$

$+$ | $-$

$\cup$ $1/2$ $\cap$

cc down $(1/2, \infty)$

cc up $(-\infty, 1/2)$

POI $(1/2, 0)$

**4. An equation of the line tangent to $y = x^3 + 3x^2 + 2$ at its point of inflection is ____.

$y' = 3x^2 + 6x$

$y'' = 6x + 6 = 0$

$x = -1$

$-$ | $+$

$\cap$ $-1$ $\cup$

$y'(-1) = 3 - 6 = -3 = m$

$y(-1) = -1 + 3 + 2 = 4$

$\boxed{y - 4 = -3(x+1)}$