Mean Value Theorem

p. 278 - 286 (4.3) # 50

If f is continuous on the interval [a, b] and differentiable on the open interval (a, b) there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Verbally SAYS: instantaneous rate of change = avg. rate of change
Graphically SAYS: tangent line is parallel to the secant line.

**(FR)1. Let f be the function given by $f(x) = x^3 - 7x + 6$. Find the number c that satisfies the conclusion of the Mean Value Theorem for f on [1, 3].

$\frac{f(3) - f(1)}{3-1} = \frac{12 - 0}{2} = 6 = 3x^2 - 7$ $f'(x) = 3x^2 - 7$

$3x^2 = 13$
$x^2 = 13/3$ $x = \pm\sqrt{13/3}$ not in domain

**(calc.) 2. Let f be the function defined by $f(x) = x + \ln x$. What is the value of c for which the instantaneous rate of change of f at $x = c$ is the same as the average rate of change of f over [1, 4]?

$\frac{f(4) - f(1)}{4-1} = \frac{4 + \ln 4 - (1)}{3} = \frac{3 + \ln 4}{3} = 1 + \frac{1}{x}$ $f'(x) = 1 + \frac{1}{x}$

$1.462097 = 1 + 1/x$ $\boxed{\text{"c"} = 2.164}$

**(FR) 3. Let f be a function that is differentiable and also $f''(x) > 0$. Using the chart below, find a positive number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

x	-1.5	-1	-0.5	0	0.5	1	1.5
f(x)	-1	-4	-6	-7	-6	-4	-1
f'(x)	-7	-5	-3	0	3	5	7

By MVT, there exists a "c" on (0, .5) where $\frac{f'(.5) - f'(0)}{.5 - 0} = f''(c) =$

$\frac{f'(.5) - f'(0)}{.5 - 0} = \frac{3 - 0}{1/2} = 6 = r$