Key

Area with respect to y

p. 441 - 445 (6.1) # 73

1. If f and g are continuous on [a,b] and $f(y) \le g(y)$ bounded by horizontal lines $y = c$ and $y = d$, then the area is found by

$$A = \int_c^d (g(y) - f(y))\, dy$$

2. If the area is Right - Left, then integrate w/respect to y.

**1. Consider the curve $y^2 = 4 + x$ and chord AB joining points A(-4, 0) and B(0, 2) on the curve. Find the area of the region R enclosed by the curve and chord AB.

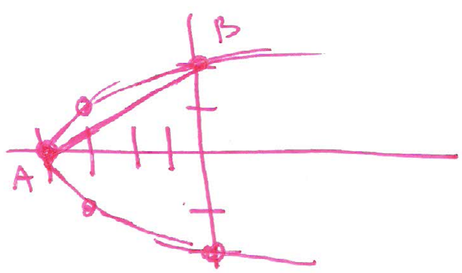
Line AB

$y = \frac{1}{2}x + 2$

$2y = x + 4$

$x = 2y - 4$

$x = y^2 - 4$



$$\int_0^2 (2y-4)-(y^2-4)\, dy$$

$$= \int_0^2 \left[2y - y^2\right] dy$$

$$= y^2 - \frac{y^3}{3} \Big|_0^2$$

$$= 4 - \frac{8}{3} - (0)$$

$$= \frac{12}{3} - \frac{8}{3} = \boxed{\frac{4}{3}}$$

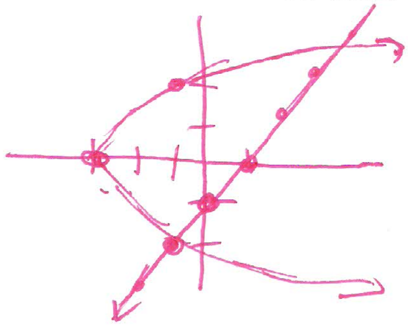
Notes on Area With Respect To y

$x = y + 1$

1. For $y = x - 1$ and $y^2 = 2x + 6$, find the area between the curves

$y^2 - 6 = 2x$ $\quad x = \frac{1}{2}y^2 - 3$

a) ~~without a calculator and~~



x	y
-3	0
-1	±2
5	±4

$\frac{1}{2}y^2 - 3 = y + 1$

$y^2 - 6 = 2y + 2$

$y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0$

$y = 4 \quad y = -2$

b) with a calculator.

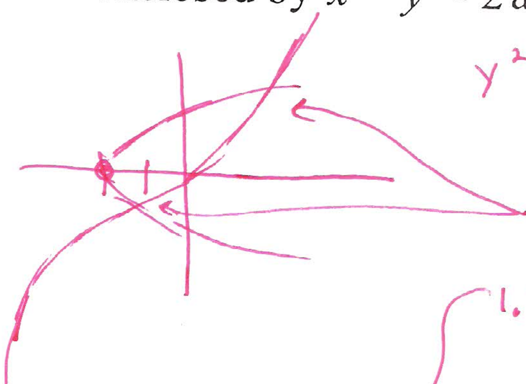
$$\int_{-2}^{4} \left[(y+1) - \left(\tfrac{1}{2}y^2 - 3\right)\right] dy = \boxed{18}$$

now you do it,

don't need to show: math 9 (x+1 − (½x² −3), x, −2, 4)

2. Use the calculator to set up how to find the area for the region enclosed by $x = y^2 - 2$ and $y = x^3$.

$y^2 - 2 = y^{1/3}$ $\quad y = \pm\sqrt{x+2}$ $\quad y = x^3$



2nd trace 5 (enter)^3

$$\int_{-1}^{1.7930037} \left[y^{1/3} - (y^2 - 2)\right] dy = \boxed{4.215}$$