Volume of Cross Sections

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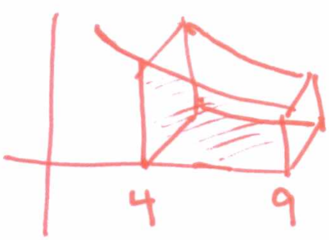
The function A represents the area of the cross section.

Perpendicular to the x-axis: $V = \int_a^b (A(x))\, dx$

Perpendicular to the y-axis: $V = \int_c^d (A(y))\, dy$

**1. Let R be the region in the 1st quadrant under $y = \frac{1}{\sqrt{x}}$ for $4 \le x \le 9$. Find the volume of the solid whose base is the region R and whose cross sections cut by planes $\perp$ to the x-axis are squares.



$V = \int_4^9 \frac{1}{\sqrt{x}} dx = \int_4^9 x^{-1/2} dx = 2x^{1/2}\Big|_4^9$

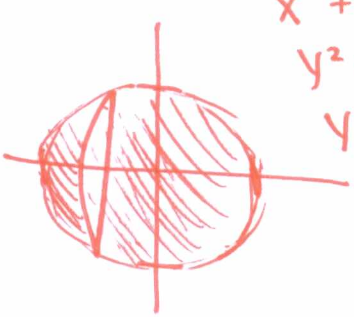
$= 2(\sqrt{9} - \sqrt{4}) = 2$

2. Find the volume of the solid whose base is enclosed by $x^2 + y^2 = 1$ and whose cross sections taken perpendicular to the base are semicircles.

$x^2 + y^2 = 1$

$y^2 = 1 - x^2$

$y = \sqrt{1 - x^2}$



$= 2\int_0^1 \frac{\pi}{2}\left(\sqrt{1-x^2}\right)^2 dx$

$= \pi \int_0^1 (1 - x^2) dx = \pi\left(x - \frac{x^3}{3}\right)\Big|_0^1$

$= \pi\left(1 - \frac{1}{3}\right) = \frac{2\pi}{3}$