Disk Method for Volume

p. 447 - 457 (6.2)

75

If the solid revolves around a horizontal axis and is flush up against the line of rotation, then the volume is found by

$$V = \int_a^b \pi r^2 \, dx.$$

If the solid revolves around a vertical axis and is flush up against the line of rotation, then the volume is found by

$$V = \int_c^d \pi r^2 \, dy.$$

Steps for Disk Method

1. Draw region.
2. Find limits.
3. Find radius to set up integral.
4. Integrate and evaluate.

**1. The region enclosed by the x-axis, the line $x = 3$, and the curve $y = \sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated? (without a calculator)

$$\int_0^3 \pi (\sqrt{x})^2 \, dx = \pi \int_0^3 x \, dx$$

$$= \pi \frac{x^2}{2} \Big|_0^3 = \frac{9}{2}\pi$$

2. Find the volume of the solid generated by revolving $x = \sqrt{1 + y}$ with $y = 3$ and $x = 0$ about the y-axis. (with and without a calculator)

$x^2 = 1 + y$

$y = x^2 - 1$

$$\pi \int_{-1}^{3} \left(\sqrt{1+y}\right)^2 dy = \pi \int_{-1}^{3} (1+y) \, dy = \pi \left[y + \frac{y^2}{2} \right]_{-1}^{3}$$

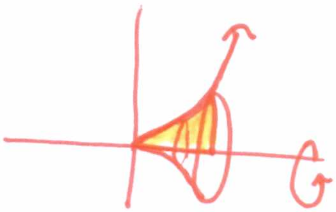
$$= \pi \left[\left(3 + \frac{9}{2}\right) - \left(-1 + \frac{1}{2}\right) \right]$$

$$= \pi \left(\frac{15}{2} + \frac{1}{2} \right)$$

$$= \boxed{8\pi}$$

Notes on Volume - Disk Method

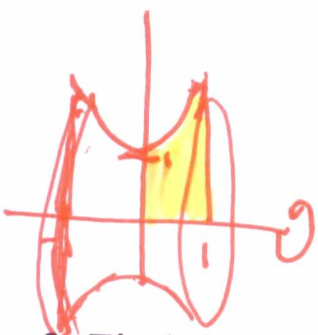
1. Find the volume of the solid generated by revolving $y = x^3$ with $y = 0$ and $x = 2$ about the x-axis.



$$\pi \int_0^2 (x^3)^2\,dx = \pi \frac{x^7}{7}\Big|_0^2$$

$$= \frac{2^7 \pi}{7} = \frac{128\pi}{7}$$

2. Find the volume bounded by the x-axis, $y = x^2 + 1$, and $x = -1$, $x = 1$ revolved about the x-axis.



$$V = 2\pi \int_0^1 (x^2+1)^2\,dx = 2\pi \int_0^1 (x^4 + 2x^2 + 1)\,dx$$

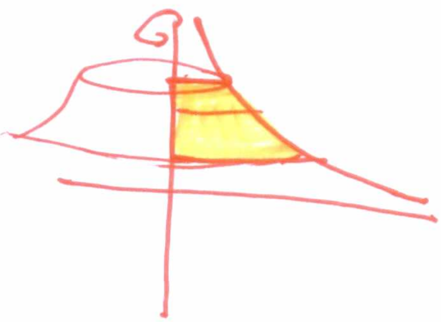
$$= 2\pi \left[\frac{x^5}{5} + \frac{2x^3}{3} + x\right]_0^1$$

$$= 2\pi \left[\frac{1}{5} + \frac{2}{3} + 1\right] = \frac{56\pi}{15}$$

3. Find the volume of the solid generated by revolving $y = \frac{1}{x}$

$y = 1$, $y = 3$, and $x = 0$ about the y-axis.

$x = \frac{1}{y}$



$$V = \pi \int_1^3 \left(\frac{1}{y}\right)^2 dy = \pi \left[\frac{y^{-1}}{-1}\right]_1^3$$

$$= \pi\left(-\frac{1}{3} - -1\right) = \frac{2\pi}{3}$$