Washer Method for Volume

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If the solid revolves around a horizontal axis and is NOT flush up against the axis of rotation, then the volume is found by

$$V = \pi \int_a^b \left(R^2 - r^2\right) dx.$$

Revolving about a vertical axis, then the volume is found by

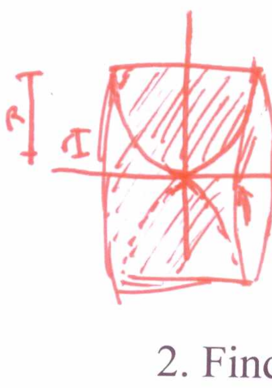
$$V = \pi \int_a^b \left(R^2 - r^2\right) dy.$$

Steps for Washer Method

1. Draw region.
2. Determine dx or dy.
3. Find limits.
4. Label big R (farthest away from axis of revolution) and little r (closest to axis of revolution) to set up the integration formula.
5. Integrate and Evaluate.

**(FR, calc.) 1. Find the volume of the solid generated by the graph bounded by $y = x^2$ and the line $y = 4$ when it is revolved about the x-axis.

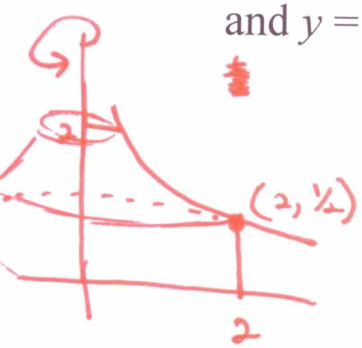
$x^2 = 4$
$x = \pm 2$



$$V = 2\pi \int_0^2 \left[(4)^2 - (x^2)^2\right] dx = 2\pi \int_0^2 (16 - x^4)\,dx = 2\pi\left[16x - \frac{x^5}{5}\right]_0^2$$

$$= 2\pi\left[32 - \frac{32}{5}\right] = 2\pi\left[\frac{160-32}{5}\right] = \frac{256}{5}\pi$$

2. Find the volume of the solid found by revolving $y = \dfrac{1}{x}$, $x = 2$, and $y = 2$ about the y-axis.

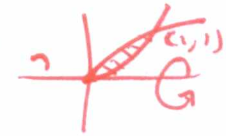


$$V = \pi \int_0^{1/2} (2)^2\,dy + \pi \int_{1/2}^{2} \left(\frac{1}{y}\right)^2 dy = \pi\left(4y\right)_0^{1/2} + \pi\left(\frac{-1}{y}\right)_{1/2}^{2}$$

$$= \left(2 + -\tfrac{1}{2} + {+2}\right)\pi = \frac{7}{2}\pi$$

Notes on Area and Volume

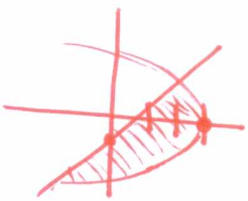
washer 1. Find the volume of the solid found by revolving $y=\sqrt{x}$ and $y=x^2$ about the x-axis.



$V=\pi\int_0^1 R^2-r^2 = \pi\int_0^1[(\sqrt{x})^2-(x^2)^2]dx \quad \pi\left[\frac{x^2}{2}-\frac{x^5}{5}\right]_0^1$

$=\pi\left[\frac{1}{2}-\frac{1}{5}\right]=\frac{3\pi}{10}$

area 2. Find the area of the region bounded by $x=3-y^2$ and $y=x-1$.



$V=\int_{-2}^{1}[(3-y^2)-(y+1)]dx=\int_{-2}^{1}(2-y^2-y)dy$

$=\left[2y-\frac{y^3}{3}-\frac{y^2}{2}\right]_{-2}^{1}=\left(2-\frac{1}{3}-\frac{1}{2}\right)-\left(-4+\frac{8}{3}-\frac{4}{2}\right)$

$=9/2$

$x=y+1$

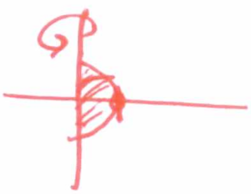
$y+1=3-y^2$

$y^2+y-2=0$

$(y+2)(y-1)=0$

$y=-2 \quad y=1$

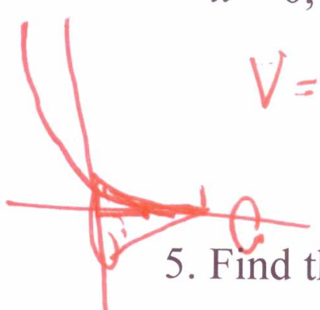
disk 3. Find the volume of the region bounded by $x=1-y^2$ and $x=0$ revolved about the y-axis.



$V=2\pi\int_0^1(1-y^2)^2=2\pi\int_0^1(1-2y^2+y^4)\,dy=2\pi\left[y-\frac{2}{3}y^3+\frac{y^5}{5}\right]_0^1$

$=2\pi\left[1-\frac{2}{3}+\frac{1}{5}\right]=\frac{16\pi}{15}$

4. Find the volume of the region bounded by $y=e^{-x}$, the x-axis, $x=0$, and $x=1$ revolved about the x-axis.



$V=\pi\int_0^1(e^{-x})^2dx=\pi\int_0^1 e^{-2x}dx=\left[\frac{\pi}{-2}e^{-2x}\right]_0^1$

$=-\frac{\pi}{2}\left[e^{-2}-e^0\right]=-\frac{\pi}{2}\left[\frac{1}{e^2}-1\right]$

5. Find the volume of the solid that results when $y=x^3$, $x=1$, and $y=0$ are revolved about the y-axis.

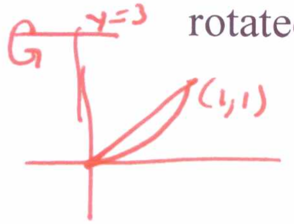
Washer

$\pi\int_0^1\left(1^2-(\sqrt[3]{y})^2\right)dy=\pi\int_0^1\left(1-y^{2/3}\right)dy$

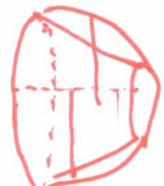
$=\pi\left[y-\frac{3}{5}y^{5/3}\right]_0^1=\pi\left[1-3/5\right]=\frac{2\pi}{5}$

Notes - Volumes Off The Axes

1. Set up the volume of the region bounded by $y = x$ and $y = x^2$ rotated about the line $y = 3$.

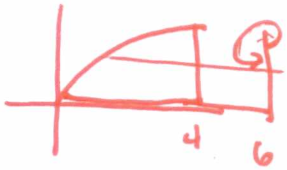


$V = \pi \int R^2 - r^2 = \pi \int_0^1 [(3-x^2)^2 - (3-x)^2]\,dx$



2. Set up the volume of the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$, revolved about the line $x = 6$.

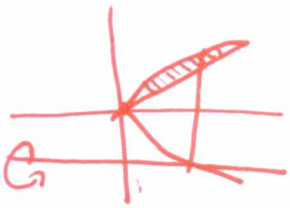
$x = y^2$ $\quad y = \sqrt{4} = 2$



$V = \pi \int_0^2 [(6-y^2)^2 - (2)^2]\,dy$

3. Set up the volume of the solid that results when the region bounded by $x = y^2$ and $x = y$ is revolved about the line $y = -1$.

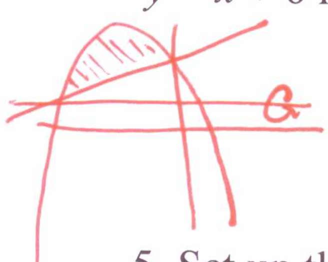
1st Quad only



$V = \pi \int_0^1 [(\sqrt{x} - {-1})^2 - (x - {-1})^2]\,dx$

4. Set up the volume of the region bounded by $y = 6 - 2x - x^2$ and $y = x + 6$ revolved about the line $y = 3$. (with calculator)

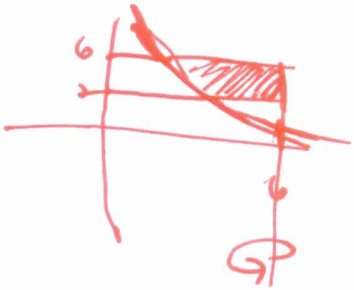
$6 - 2x - x^2 = x + 6$
$x^2 + 3x = 0$
$x(x+3)$
$x = 0, -3$

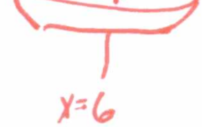


$V = \pi \int_{-3}^{0} [(6 - 2x - x^2 - 3)^2 - (x + 6 - 3)^2]\,dx$

5. Set up the volume of the region bounded by $xy = 6$, $y = 2$, $y = 6$, $x = 6$ revolved about the line $x = 6$.

$y = \frac{6}{x}$





$V = \pi \int_2^6 \left(6 - \frac{6}{y}\right)^2 dy$