

Average Value of a Function

p. 467 - 469 (6.4)

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If f is integrable on $[a, b]$, its **average (mean) value** on $[a, b]$ is

$$\frac{1}{b-a} \int_a^b (f(x)) dx.$$

a)
$$\frac{\int_a^b (f(x)) dx}{b-a} = \frac{\text{area}}{\text{width}} f(c) = \text{average height}$$


b) Finding "c" is where the average height occurs

c) Average value = average velocity

**1. Find the average value of $y = x^2 \sqrt{x^3 + 1}$ on the interval $[0, 2]$.

$$\begin{aligned} \frac{1}{2-0} \int_0^2 x^2 \sqrt{x^3+1} dx & \quad \begin{array}{l} u = x^3+1 \\ du = 3x^2 dx \\ dx = \frac{du}{3x^2} \end{array} \rightarrow \frac{1}{6} \left[\frac{2}{3} \cup \frac{3}{2} \right]_a^b = \frac{1}{9} (x^3+1)^{\frac{3}{2}} \Big|_0^2 \\ & = \frac{1}{2} \int_a^b x^2 \sqrt{u} \frac{du}{3x^2} \\ & = \frac{1}{6} \int_a^b u^{\frac{1}{2}} du \end{aligned}$$

$= \frac{1}{9} [(9)^{\frac{3}{2}} - 1] = 3 - \frac{1}{9} = \frac{26}{9}$



2. Find the average value of $f(x) = \sin(x)$ over $[0, \pi]$.

$$\begin{aligned} \frac{1}{\pi-0} \int_0^\pi \sin x dx & = \left. \frac{-\cos x}{\pi} \right|_0^\pi = \frac{1}{\pi} [-\cos \pi + \cos 0] \\ & = \frac{1}{\pi} [1 + 1] = \frac{2}{\pi} \end{aligned}$$

**calc.(FR) 3. A function f , $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3 \sin\left(\frac{7t}{40}\right)$, is used

to model the velocity of a plane in miles per minute. According to this model, what is the average velocity of the plane for $0 \leq t \leq 40$?

$$\frac{1}{40-0} \int_0^{40} f(t) dt = \text{~~5.9162~~ 5.9162}$$