Mean Value Theorem for Integrals

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If f is continuous on [a,b] , then at some point "c" in [a,b]

$$f(c) = \frac{1}{b - a}\int_a^b (f(x))\, dx$$

$$\text{height} = \left(\frac{1}{\text{width}}\right)(\text{area})$$

***Recall Mean Value Theorem $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

That is, there is a rectangle whose area is precisely equal to the area of the region under the curve.

Using the MVT for integrals, find the value of "c".

1. $f(x) = \dfrac{4}{x^2}$ for [1,4]

$$f(c) = \frac{1}{4-1}\int_1^4 \frac{4}{x^2}dx = \frac{4}{3}\int_1^4 x^{-2}dx = \frac{-4}{3}x^{-1}\Big|_1^4 = -\frac{4}{3}\left[\frac{1}{x}\right]_1^4 = -\frac{4}{3}\left[\frac{1}{4} - 1\right] = 1$$

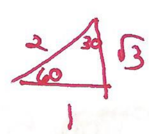
$$f(c) = 1 \Rightarrow \frac{4}{x^2} = 1 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$$ (but -2 not in domain)

so $\boxed{x = 2}$

2. $f(x) = \cos(x)$ over $\left[\dfrac{-\pi}{3}, \dfrac{\pi}{3}\right]$

$$\frac{1}{\frac{\pi}{3} + \frac{\pi}{3}}\int_{-\pi/3}^{\pi/3} \cos x\, dx = \frac{1}{\frac{2\pi}{3}}\int_{-\pi/3}^{\pi/3} \cos x\, dx = \frac{3}{2\pi}\Big[\sin x\Big]_{-\pi/3}^{\pi/3}$$

$$= \frac{3}{2\pi}\left[\sin\frac{\pi}{3} - \sin\frac{-\pi}{3}\right]$$



$$= \frac{3}{2\pi}\left[\frac{\sqrt{3}}{2} - \left(\frac{-\sqrt{3}}{2}\right)\right] = \frac{3}{2\pi}\left[\frac{\cancel{2}\sqrt{3}}{\cancel{2}}\right] = \frac{3\sqrt{3}}{2\pi}$$

$$\cos x = \frac{3\sqrt{3}}{2\pi}$$

$$x = \cos^{-1}\left(\frac{3\sqrt{3}}{2\pi}\right)$$

$\boxed{x = .597}$